Capabilities/Issues in Computational Contact Mechanics

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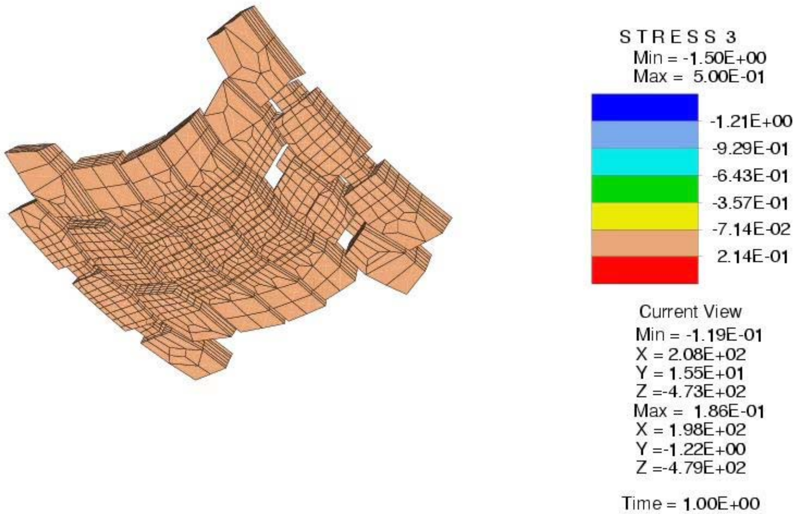
Goal of our Research

Finite Element algorithms for large deformation, deformable-to-deformable contact in quasistatic and implicit dynamic analysis, suitable for:

- High fidelity prediction of frictional behaviors in a wide variety of physical settings (stick slip behavior in forming operations; microslip damping phenomena giving rise to structural damping; self-contact and frictional dissipation in tire rolling)
- Accurate treatment of impact phenomena, with careful attention in particular paid to conservation/dissipation of momenta and energy
- Increasingly, incorporation of tribological complexity in our capabilities for contact simulation (including lubrication)

An example from our collaboration with Michelin:

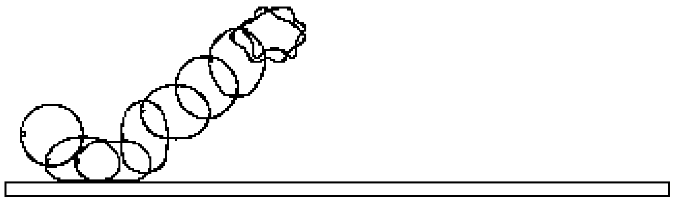


Some Past Efforts Relevant to this Goal

Energy-Momentum Formulation of Impact Interaction

Motivation: many traditional finite element integrators for impact interaction are only linearly stable, and in nonlinear impact calculations can readily produce unstable behavior

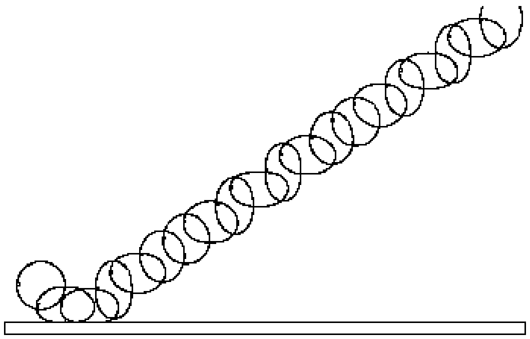
Example:HHT integration of ring impact



Idea: develop algorithms for impact that explicitly conserve energy (when appropriate), as well as linear and angular momentum

Accomplishments of this work (see Laursen & Chawla [1997]; Chawla & Laursen [1998]; Laursen & Love [2002]; Love & Laursen [2003]):

- Stable algorithms for conservative (frictionless) contact without introduction of nonphysical damping
- Introduction of surface and bulk dissipation (inelasticity) in a manner consistent with underlying thermodynamics
- New notions of temporal accuracy, and corresponding implementations,within an energy-momentum framework



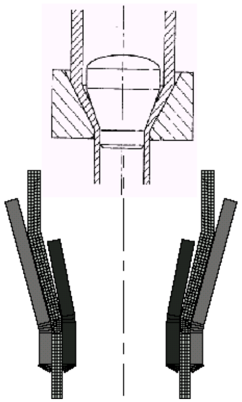
Stable, energy-momentum solution

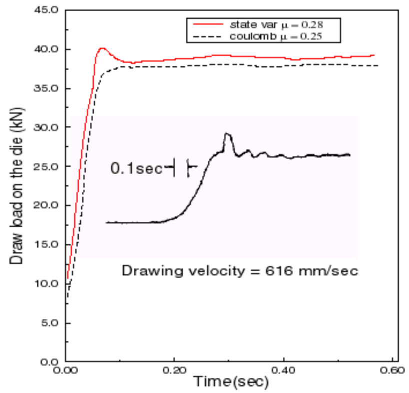
Past Efforts (Cont.)

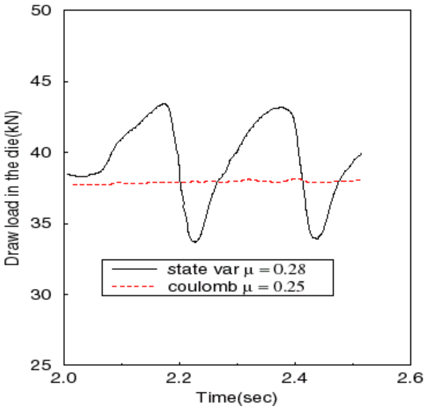
Complexity on Interfaces: Multifield Coupling and Tribological Modeling

Motivation: many applications demand sophisticated interface constitutive laws to describe observed phenomena

Example: chatter instabilities in drawing applications (Oancea and Laursen [1997, 1998]

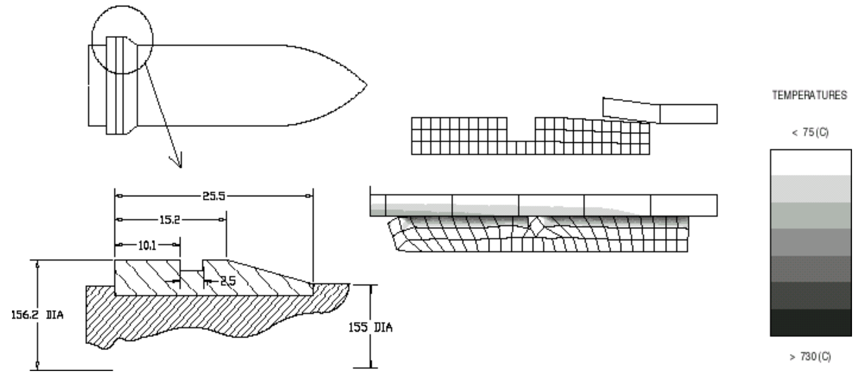






Accomplishments of our research:

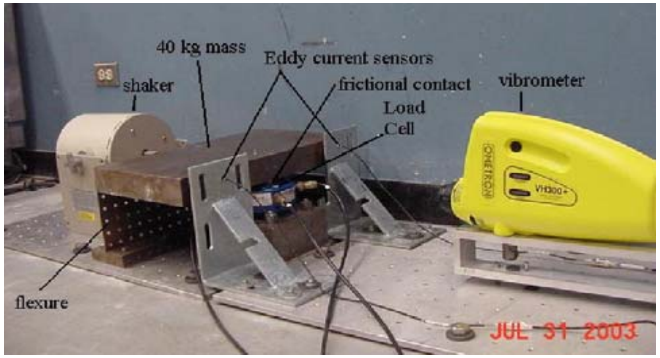
- Theoretical framework enabling stable extension of mechanical descriptions to encompass thermomechanical coupling, enabling simulation of frictional heating, thermal softening (as in shell firing simulation, right)
- Implementations of frictional rate dependence, enabling simulation of unstable slip (see above)

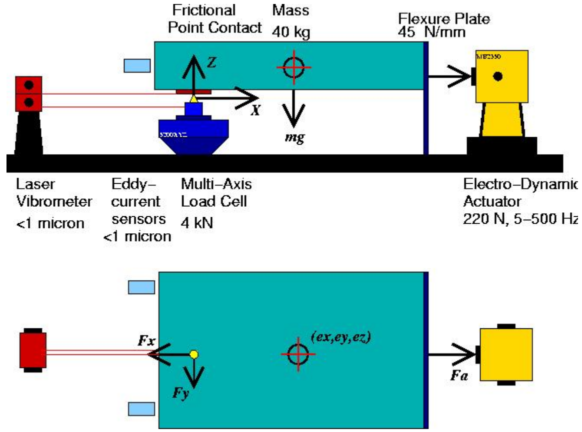


Past Efforts (Cont.)

Microslip Damping/Hysteresis Prediction without Phenomenology

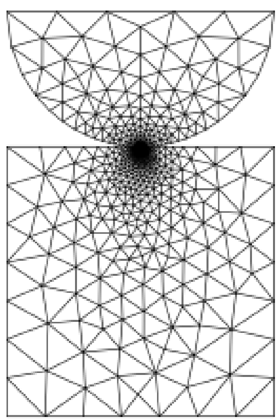
Motivation: Many structural damping applications are limited by reliance on phenomenological results, in which distinctions between bulk compliance and surface effects cannot be drawn

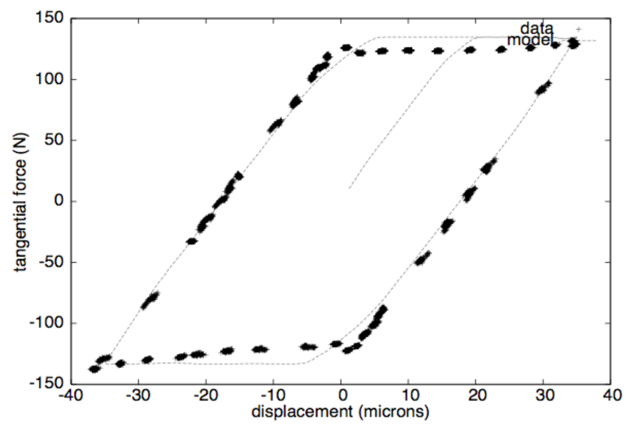




Recent Accomplishments (Greer [2004]):

- Good representation of hysteretic behavior in finite element models of Mindlin-type experiments (friction law requires two inputs which are readily measured experimentally: mu and an interface stiffness)





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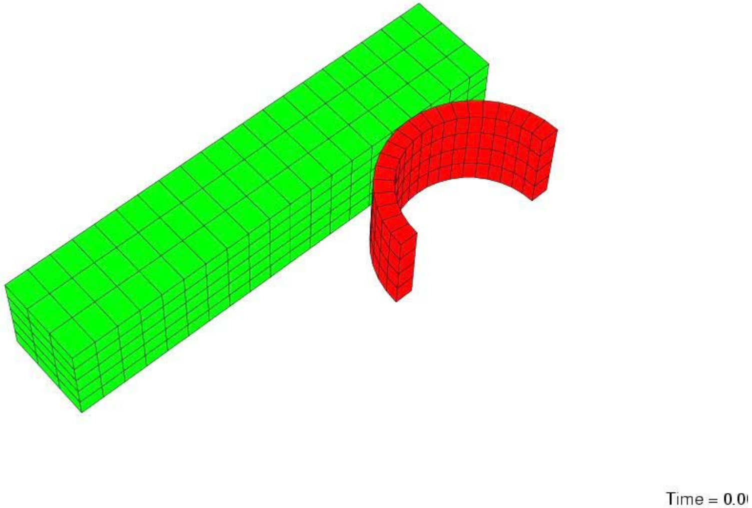
Some Preliminary Observations

From this (very brief!) examination of the types of problems we are interested in, we infer some needs that have driven our research in the past few years:

- We would like methods of contact analysis that are accurate: (ideally, presence of contact should not degrade spatial convergence rates expected from underlying finite element methods)
- We need numerical robustness (particulaly in large sliding and/or deformation applications, where connectivity continually changes throughout the simulation)
- We want broad applicability: two and three dimensions, with a variety of material models, with and without friction, extendible to tribologically complex settings (including rate dependence, anisotropy, lubrication)



Finite Element Formulation

If we approximate this system by imposing a finite element grid, we end up with an equation system of the form

$$\boldsymbol{M}\ddot{\boldsymbol{d}}(t) + \boldsymbol{F}^{int}(\boldsymbol{d}(t)) + \boldsymbol{F}_c(\boldsymbol{d}(t)) = \boldsymbol{F}^{ext}(t)$$

where

- $\boldsymbol{M}$ is the mass matrix
- $\boldsymbol{F}^{int}$ is the internal force vector, a generally nonlinear function of $\boldsymbol{d}$
- $\boldsymbol{F}_c$ is the contact force vector, subject to the aforementioned restrictions
- $\boldsymbol{F}^{ext}$ is the imposed external loading

In solving such a system, several challenges manifest themselves:

- Nonlinear equation solving (subject to nonsmooth constraints)
- Potential ill-conditioning
- Stability problems in dynamics
- Detection of contact (i.e., searching)
- Spatial discretization of contact constraints, and its effect on the results obtained

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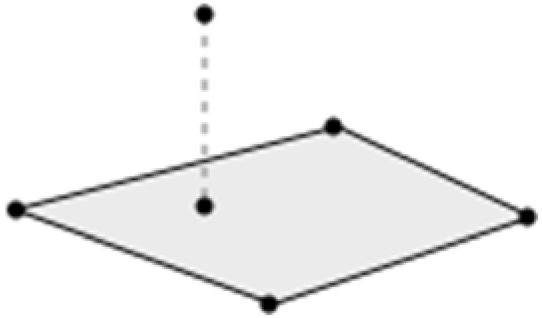
Traditional Approach to Contact Mechanics in Finite Element Analysis

"Node to Surface" Contact, where constraints are imposed for nodes with respect to opposing element surfaces:



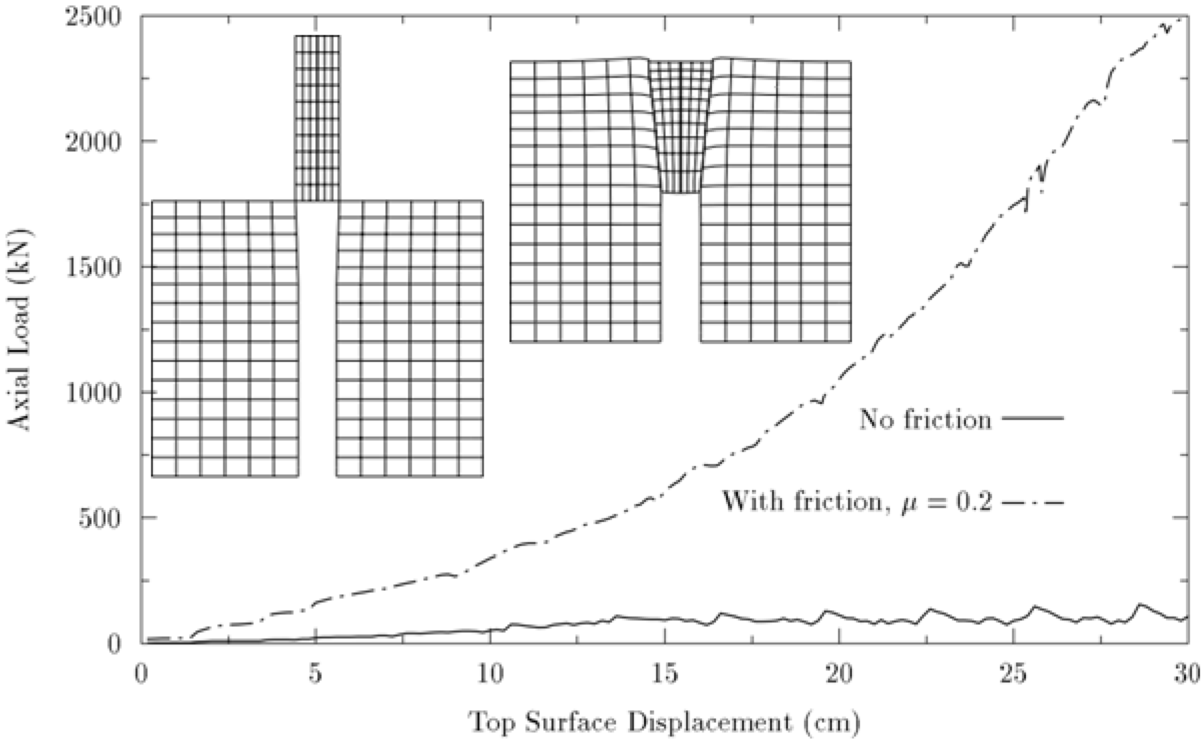
We can think of this as sort of a collocation approach, with the collocation points being the nodes of one side (or both)

Some problems with this approach are evident

- When nodes slide across element boundaries, nonsmoothnesses are introduced
- Low order solutions are not admitted by the formulation
- Accurate contact traction recovery is difficult
- Non-conforming FE approximation→suboptimal convergence

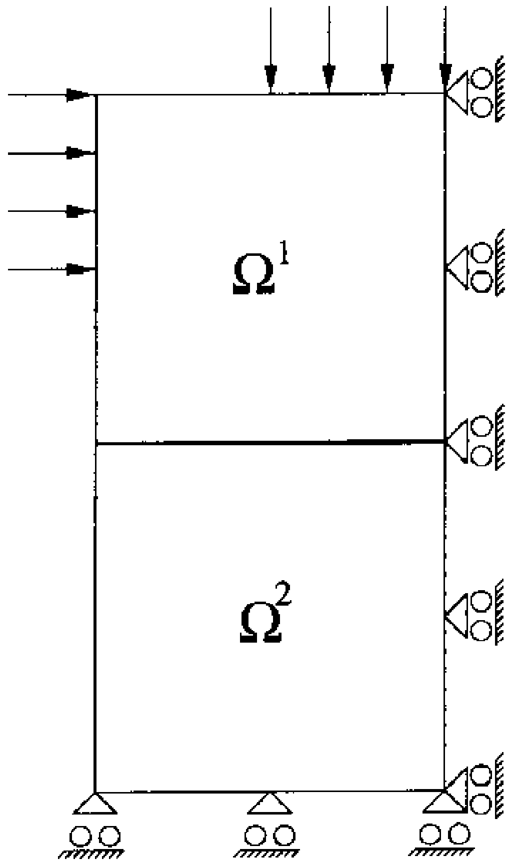
One Limitation of Traditional Approach: Nonsmoothnesses from Faceted Geometries

Because node to surface schemes enforce constraints with respect to a faceted geometry, both convergence difficulties and nonphysical results are to be expected in deformable interface contact problems.

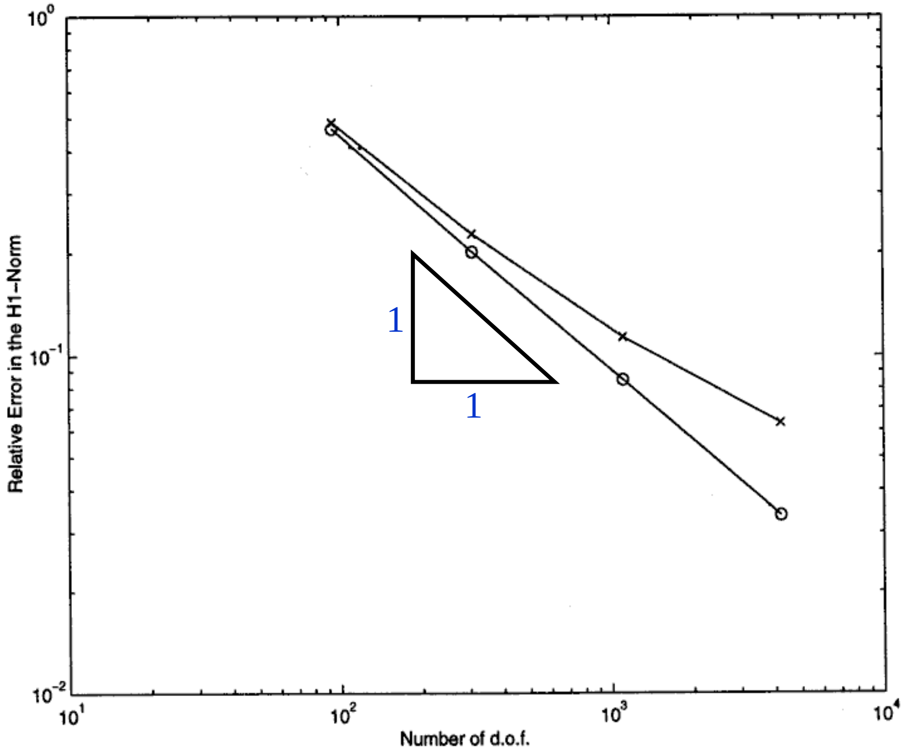


Another Limitation: Convergence is Demonstrably Degraded in Node to Surface Treatments

Convergence Study due to Hild [2000]



Test problem



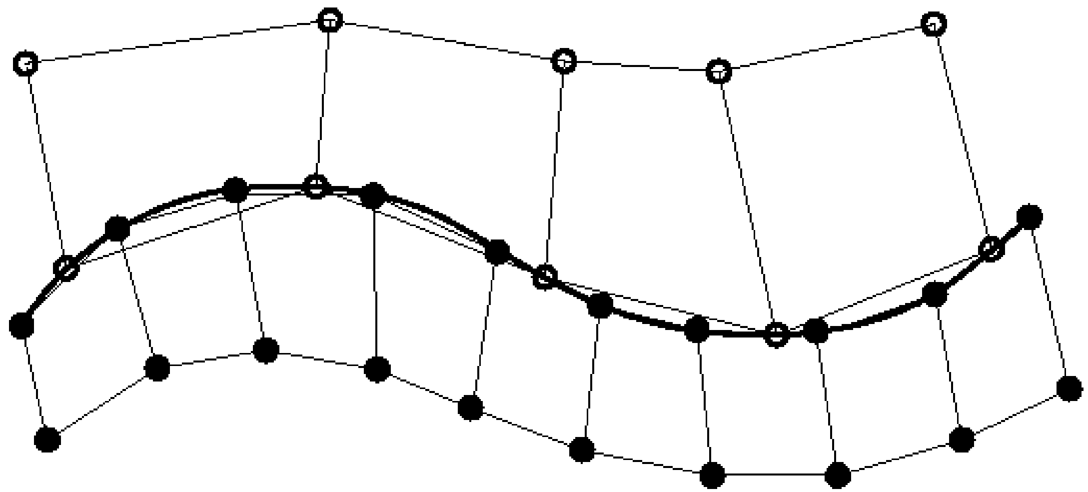
○ Mortar Method

× Node-to-Segment

A (Relatively) New Approach: Mortar-Finite Element Methods

The "node to surface" contact formulation is avoided by considering an integral formulation of contact conditions

- To demonstrate this idea, consider the mesh tying problem as a template
- This is a problem of great practical importance: dissimilar discretizations of the same curve give rise not only to "contact-like" geometries, but also to areas of gaps and overlaps between surfaces to be joined



Basic Idea of Mortar Concept

For the tying problem, we enforce compatibility of a least squares projection of one displacement field with that of the opposing surface.

Key ideas:

- Integral representation of displacement continuity

$$0 = \int_{\Gamma_c^{(1)^h}} \boldsymbol{q}^h \cdot (\boldsymbol{U}^{(1)^h} - \boldsymbol{U}^{(2)^h})\, d\Gamma$$

where the multipliers (tractions) $\boldsymbol{\lambda}^h$ are interpolated via

$$\boldsymbol{\lambda}^h = \sum_{A=1}^{nnod(1)} \boldsymbol{\lambda}_A N_A$$

- Combination of the above leads to constraints c_A of the form

$$0 = \boldsymbol{c}_A = \int_{\Gamma_c^{(1)^h}} N_A \left(\sum_{B=1}^{nnod(1)} \boldsymbol{U}_B N_B - \sum_{\hat{B}=1}^{nnod(2)} \boldsymbol{U}_{\hat{B}} N_{\hat{B}} \right) d\Gamma$$

involving inner products of shape functions, i.e. $M_{AB} = \int_{\Gamma_c^{(1)^h}} N_A N_A \, d\Gamma$

We Use this Idea for Discretization of Contact Interaction, but Recognize that Mortar Integrals must Depend on Deformation

Contact virtual work:

$$G^{cm}(\boldsymbol{\varphi}^h, \overset{*}{\boldsymbol{\varphi}}^h) := -\int_{\gamma_c^{(1)^h}} \boldsymbol{\lambda}^h(\boldsymbol{X}) \cdot \left(\overset{*}{\boldsymbol{\varphi}}^{(1)^h}(\boldsymbol{X}) - \overset{*}{\boldsymbol{\varphi}}^{(2)^h}(\bar{\boldsymbol{Y}}) \right) d\gamma$$

- The discretized contact traction and deformation fields are defined as:

$$\boldsymbol{\lambda}^h(\boldsymbol{X}) = \sum_{A=1}^{n_s} N_A^{(1)} \left(\boldsymbol{\xi}^{(1)}(\boldsymbol{X}) \right) \boldsymbol{\lambda}_A \qquad \boldsymbol{\varphi}^{(1)^h}(\boldsymbol{X}) = \sum_{D=1}^{n_s} N_D^{(1)} \left(\boldsymbol{\xi}^{(1)}(\boldsymbol{X}) \right) \boldsymbol{\varphi}_D^{(1)}$$

(etc. for the other fields)

- Finally, the discretized contact virtual work is

$$G^{cm}(\boldsymbol{\varphi}^h, \overset{*}{\boldsymbol{\varphi}}^h) = -\sum_A \sum_B \sum_C \boldsymbol{\lambda}_A \cdot \left[n_{AB}^{(1)} \overset{*}{\boldsymbol{\varphi}}_B^{(1)} - n_{AC}^{(2)} \overset{*}{\boldsymbol{\varphi}}_C^{(2)} \right]$$

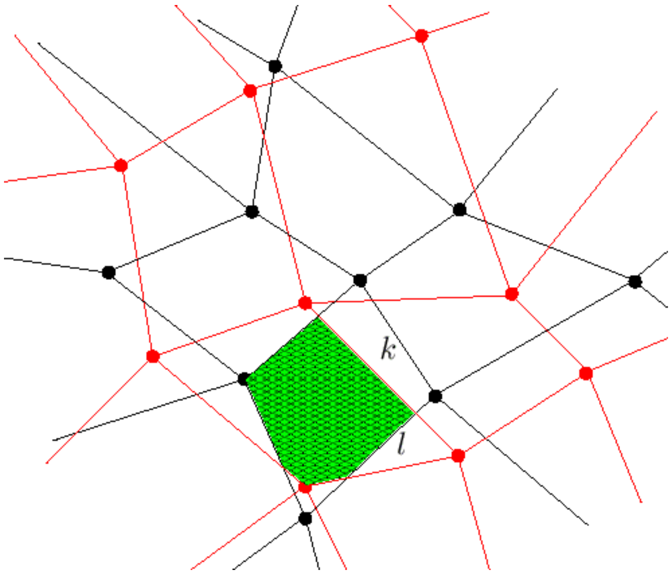
- Where the mortar integrals are now computed in the current configuration

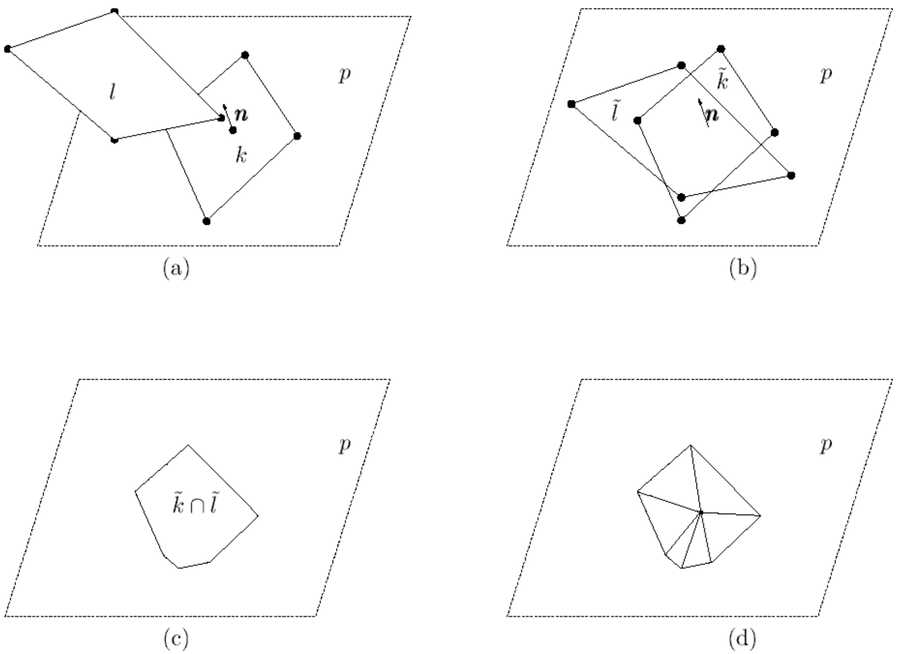
$$n_{AB}^{(1)} = \int_{\gamma^{(1)^h}} N_A^{(1)} \left(\boldsymbol{\xi}^{(1)}(\boldsymbol{X}) \right) N_B^{(1)} \left(\boldsymbol{\xi}^{(1)}(\boldsymbol{X}) \right) d\gamma$$

$$n_{AC}^{(2)} = \int_{\gamma^{(1)^h}} N_A^{(1)} \left(\boldsymbol{\xi}^{(1)}(\boldsymbol{X}) \right) N_C^{(2)} \left(\boldsymbol{\xi}^{(2)} \left(\bar{\boldsymbol{Y}} \left(\boldsymbol{X} \right) \right) \right) d\gamma$$

Computation of mortar integrals: three dimensions

Computation of $n_{AB}^{(1)}$ and $n_{AC}^{(2)}$ in three dimensions uses extension of same idea, but algorithm is necessarily much more involved (see Puso and Laursen [2003])





Key Ideas:

(1) Define the flat projection surface *p*

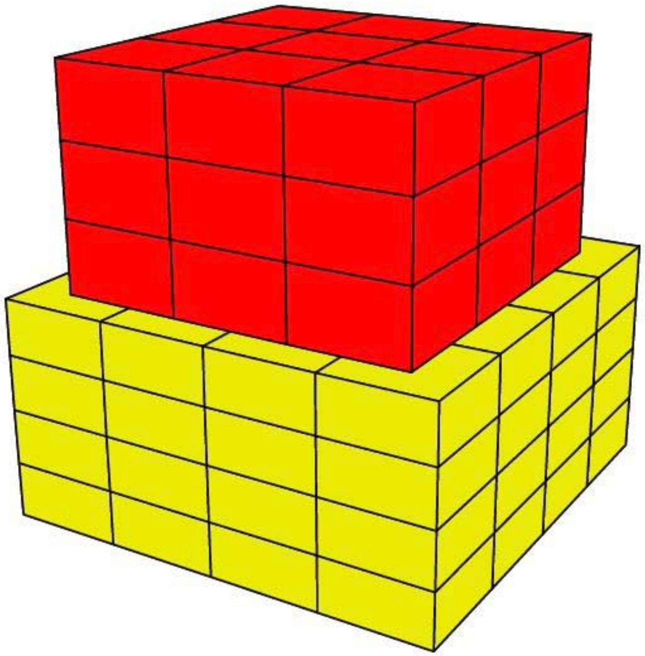
(2) Project slave and master elements onto the surface *p*

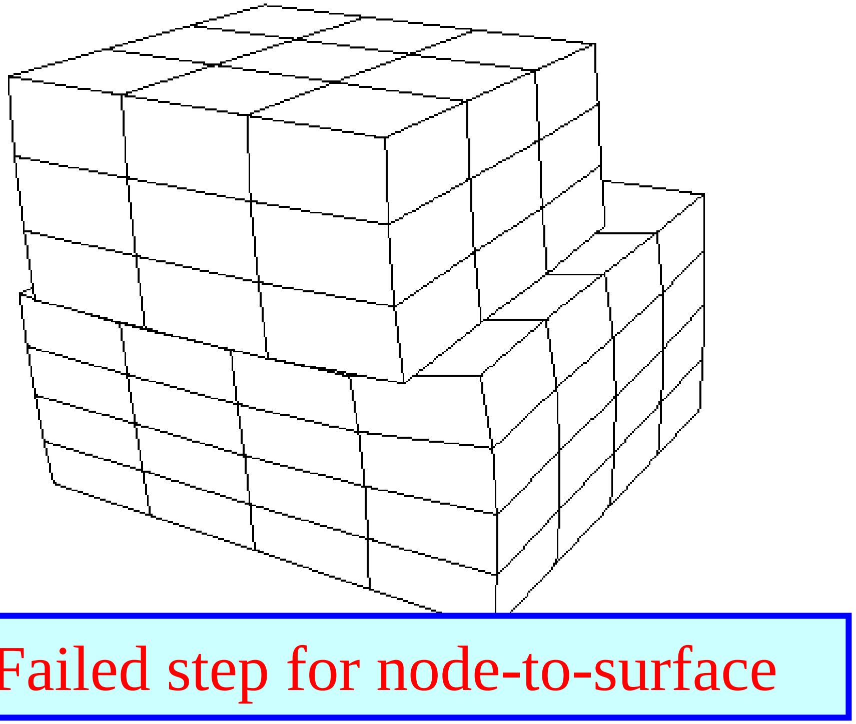
(3) Find the intersection of the projected polygons

(4) Divide into triangles to perform numerical integration

Some Numerical Examples

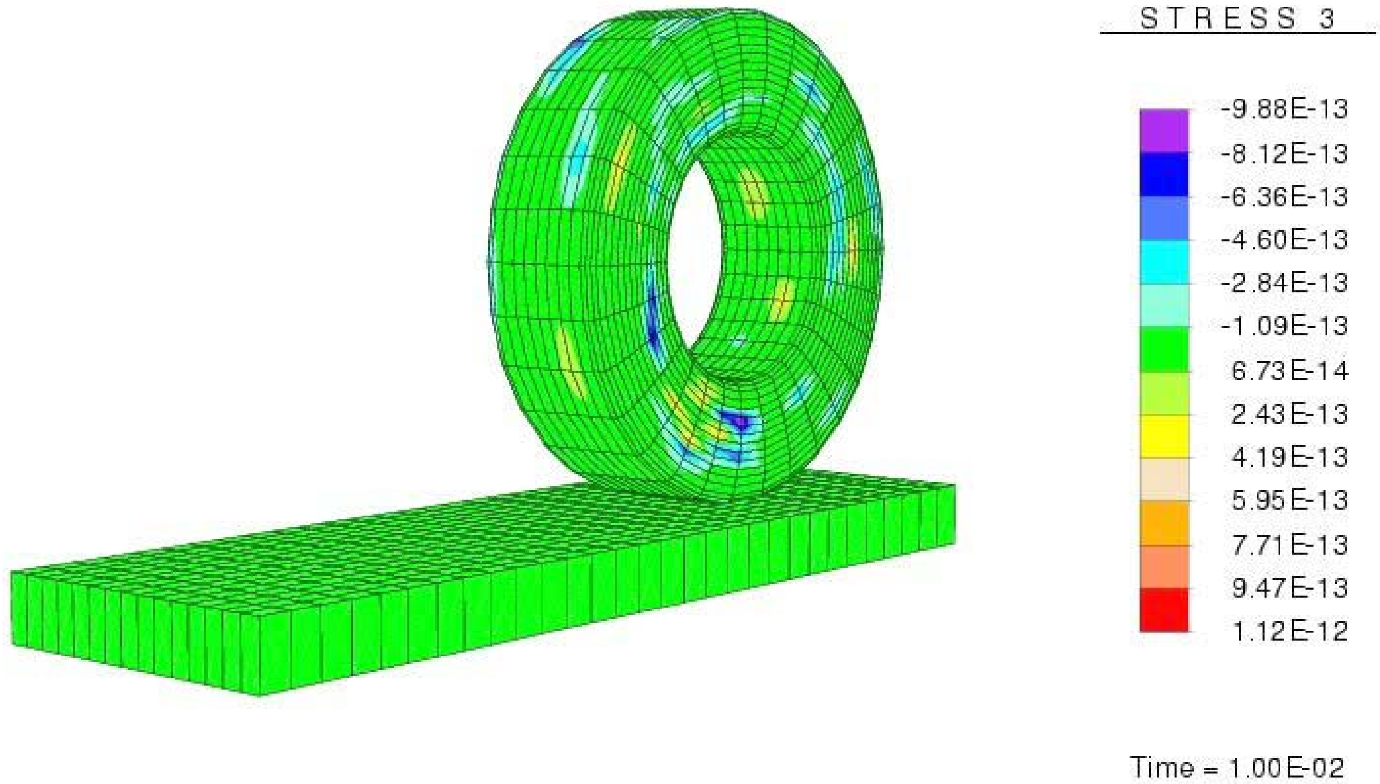
This simple 3D problem demonstrates robustness when nodes leave contact, resulting from nonlocal constraint definitions





- Sliding and pressing the upper block, part of the upper block slides out of the lower block
- The node-to-surface fails at t=0.29

An Industrial "Toy" problem for Michelin (self-contact occurs inside the tire also)



Current State of the Art

In computational contact mechanics, the past few years have seen several advances

- Energetically consistent algorithms for contact mechanics
- Some incorporation of new constitutive models for friction (including thermomechanical contact)
- New classes of algorithms giving much greater numerical accuracy and robustness (mortar methods)
  - As D. Segalman mentioned yesterday, this is not to be taken for granted (physics issues aside)

Challenges:

- Mesh density requirements for good resolution of lubrication, dry friction damping
- Multiscale
- Is there a chance of avoiding explicit gridding of interfaces altogether?

We Think Yes

A New Direction (joint with J. Dolbow): XFEM treatment of interfaces

Polycrystalline elastic beam bending: