Estimating the Degree of Nonlinearity in Transient Responses with Zeroed Early-Time Fast Fourier Transforms

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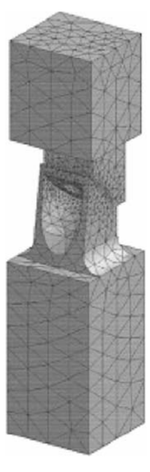
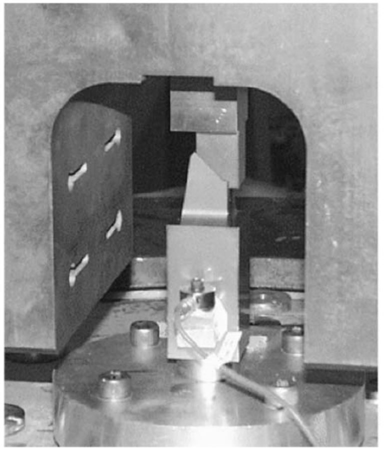
Introduction

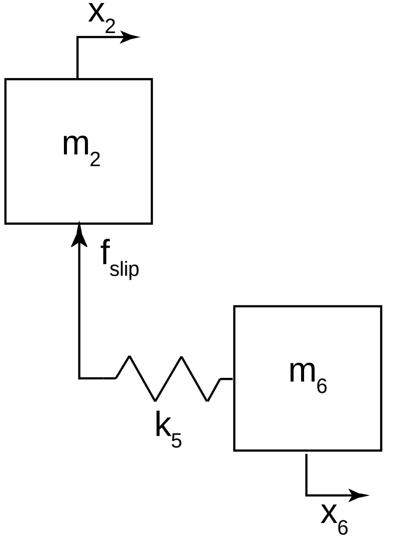
- Challenge
  - Detect and characterize short duration nonlinearity in transient response data from relatively high order systems.
- Proposed Tools and Theory
  - Zeroed Early-time FFT (ZEFFT)
  - Backwards Extrapolation for Nonlinearity Detection (BEND)

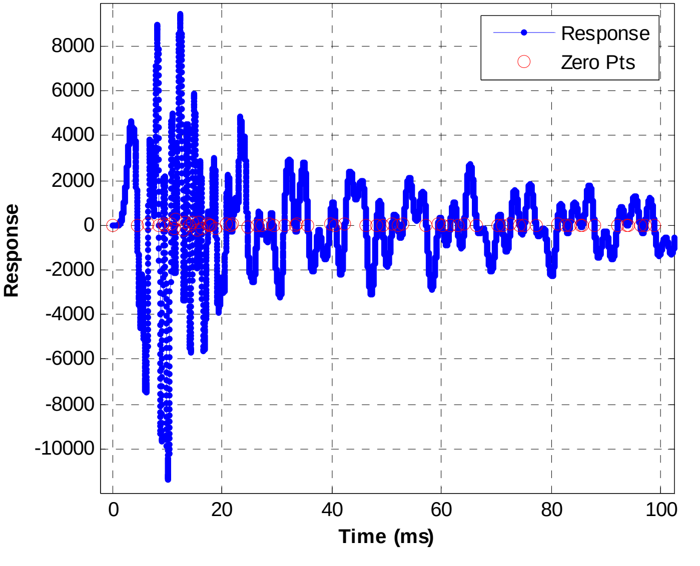
Challenge

- Some systems with bolted joints respond to impulses nonlinearly in the first few cycles followed by a linear decay.









Challenge (2)

- Full nonlinear system identification of structures with joints is extremely difficult.
  - Moderate to High order systems are currently beyond the reach of state of the art nonlinear system identification algorithms
  - Time-frequency methods may not have sufficient resolution for the responses of interest. (i.e. Spectrogram, Wavelet, Choi-Williams, Hilbert-Huang…)
- What can we do then?

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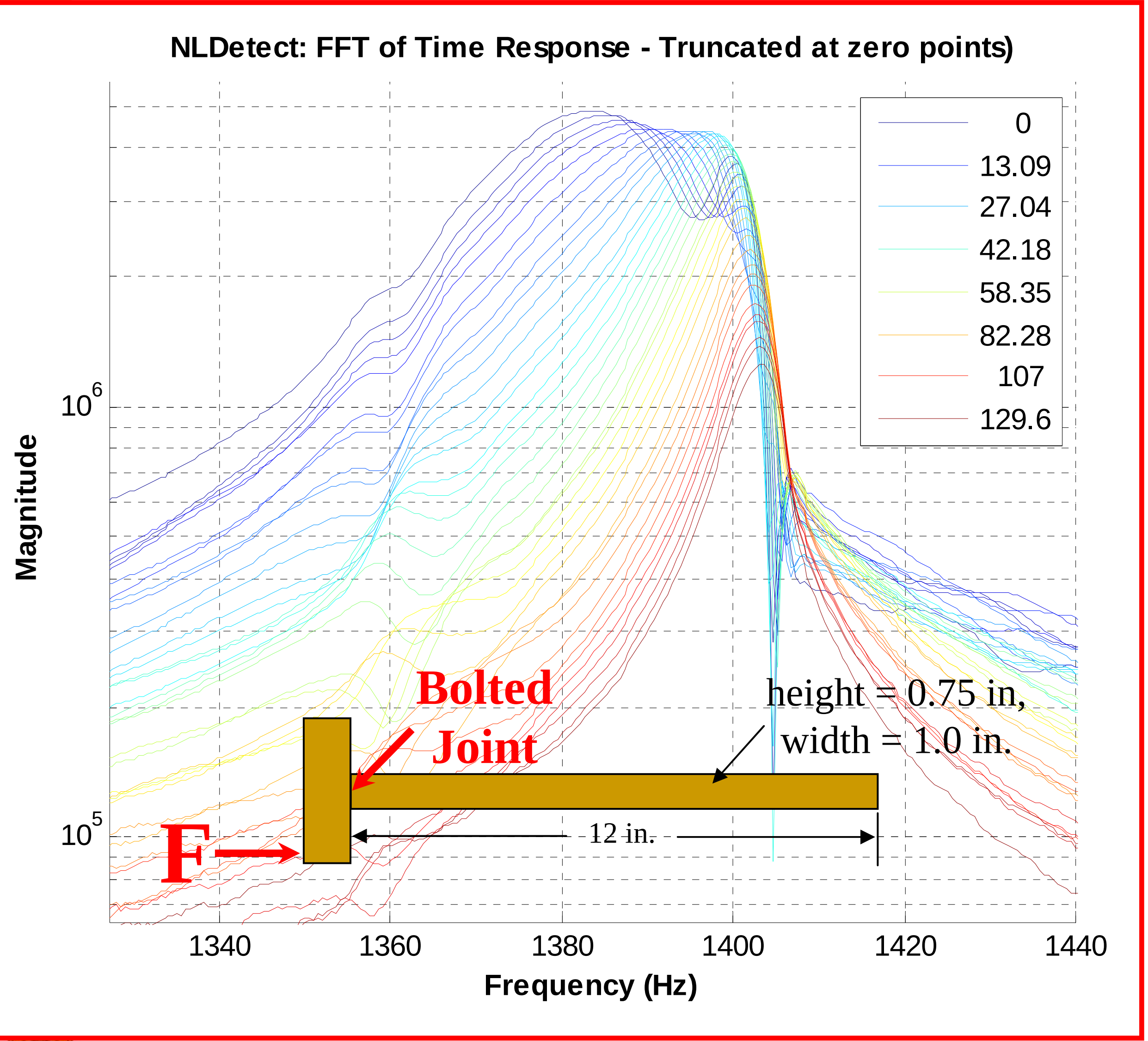
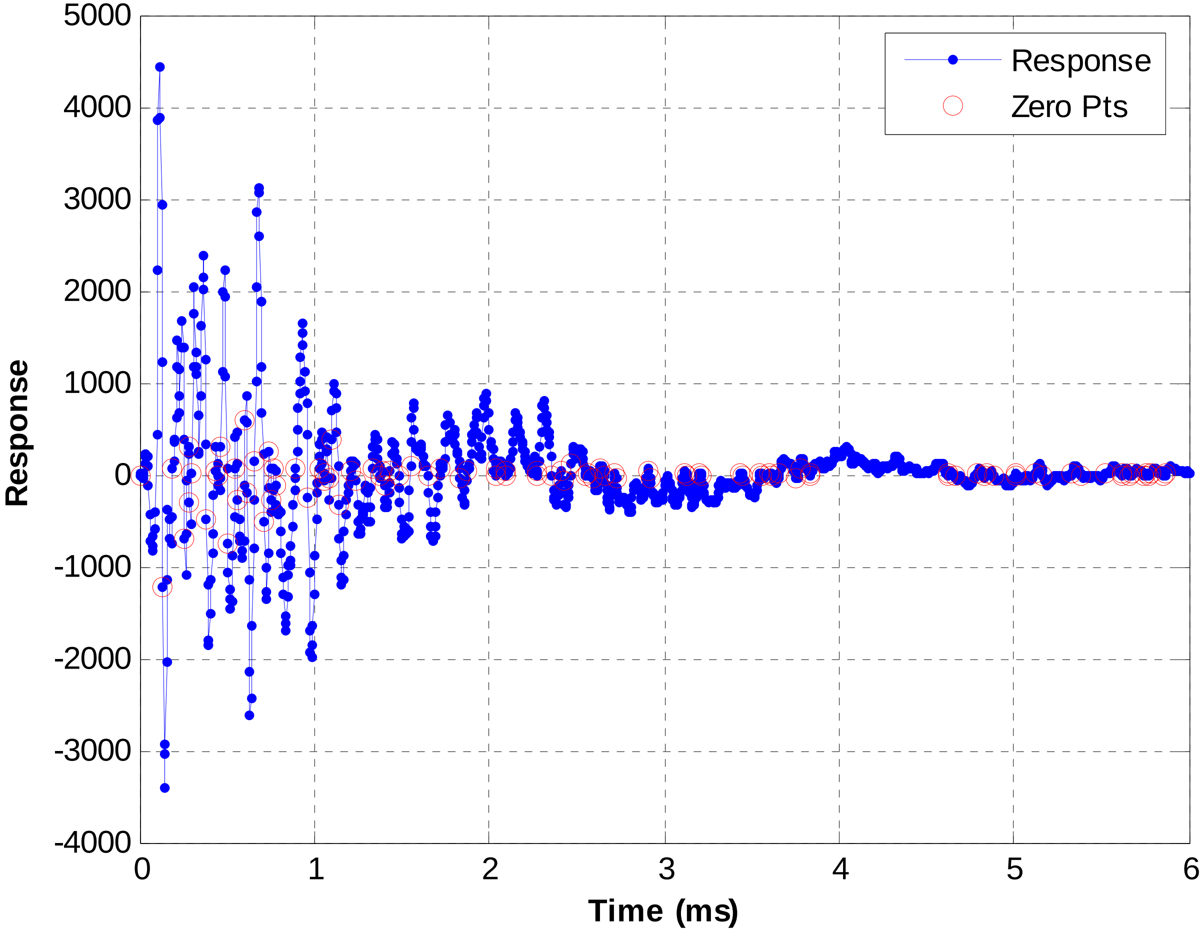
Zeroed Early-time FFT



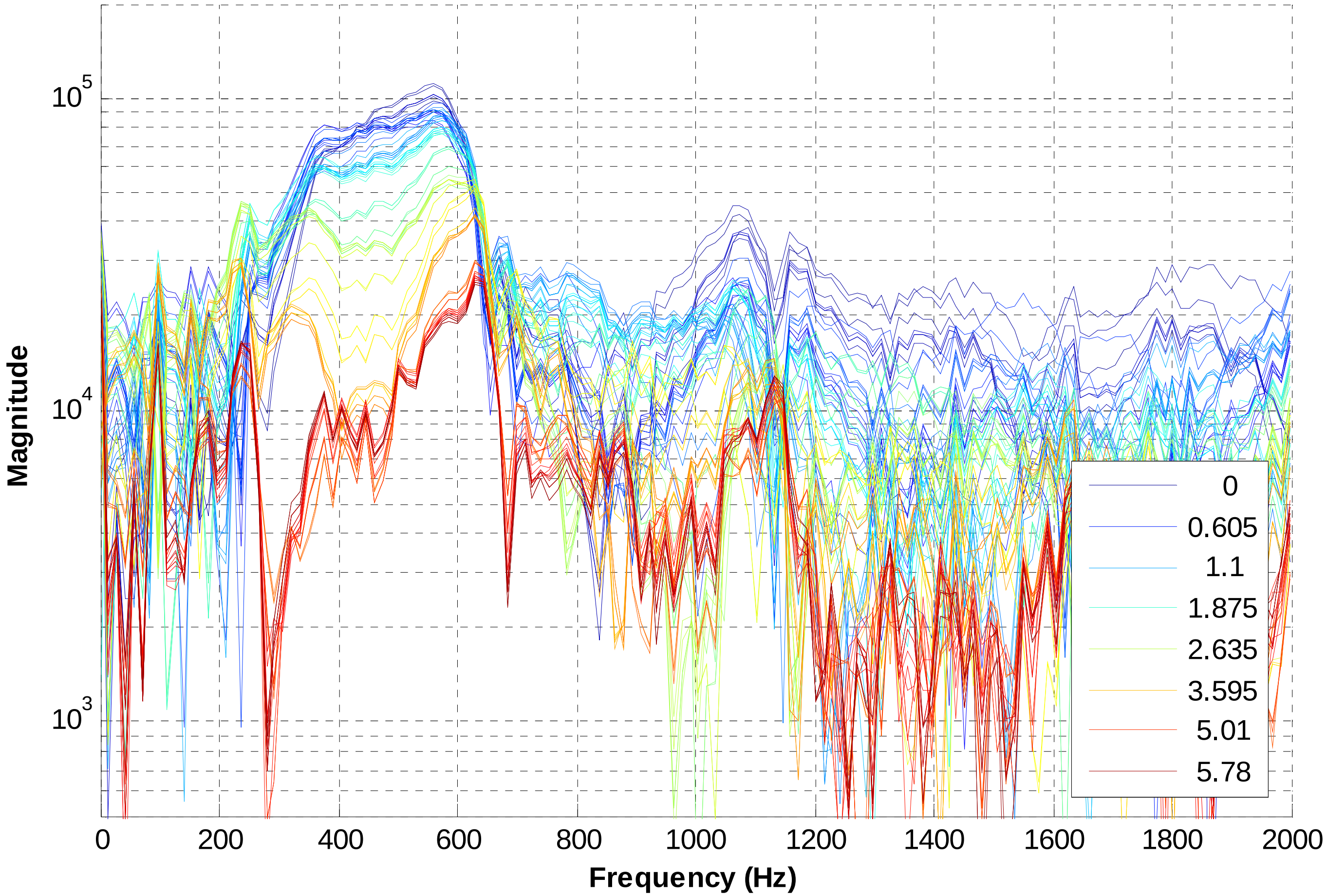
- Nonlinearity is assumed to be active at high amplitudes and inactive at lower amplitudes
- The response then becomes more linear as more of the initial nonlinear response is nullified.
- Impulse responses with initial segments of varying length set to zero are compared in the frequency domain.
- The nonzero portion of each impulse response begins at a point in which the response is near zero.

Experimental Shock Data: Time Response



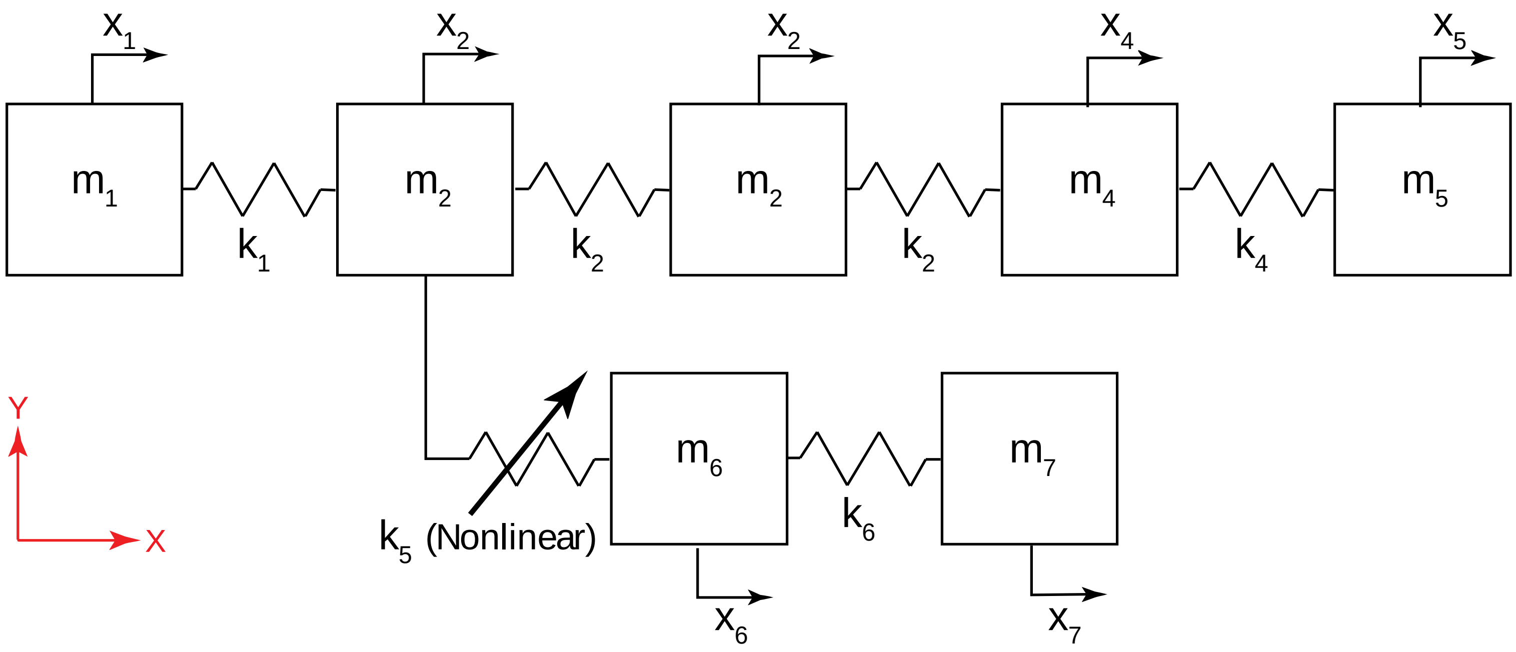
- Response of a complex structure with a bolted joint to an impulsive force.

Shock Data: ZEFFTs



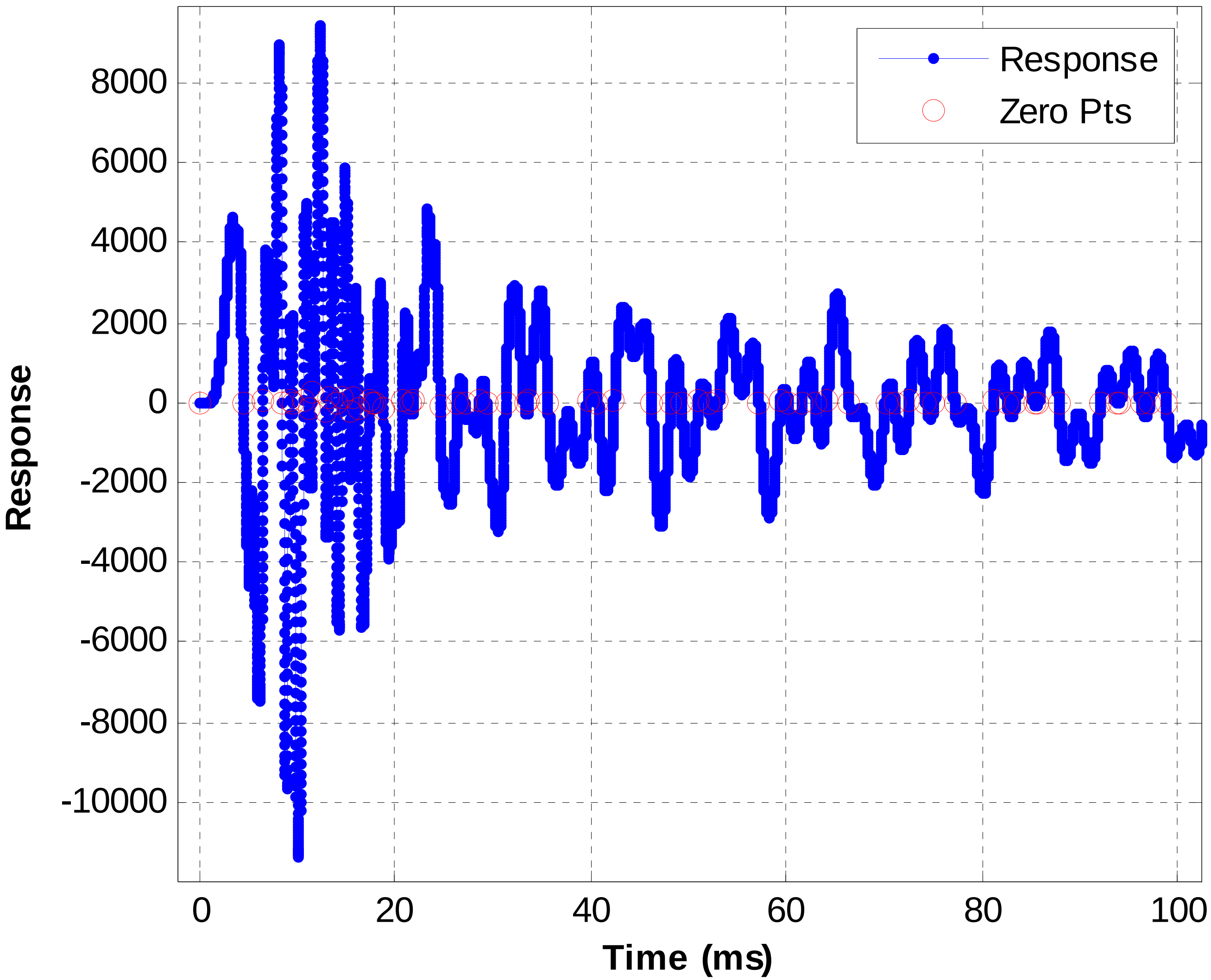
- Response contains a large, broad peak between 375 and 600 Hz at early times.
- Response appears to be very noisy above 800 Hz. This could be due in part to harmonics of the lower frequency modes.

Analytical Example



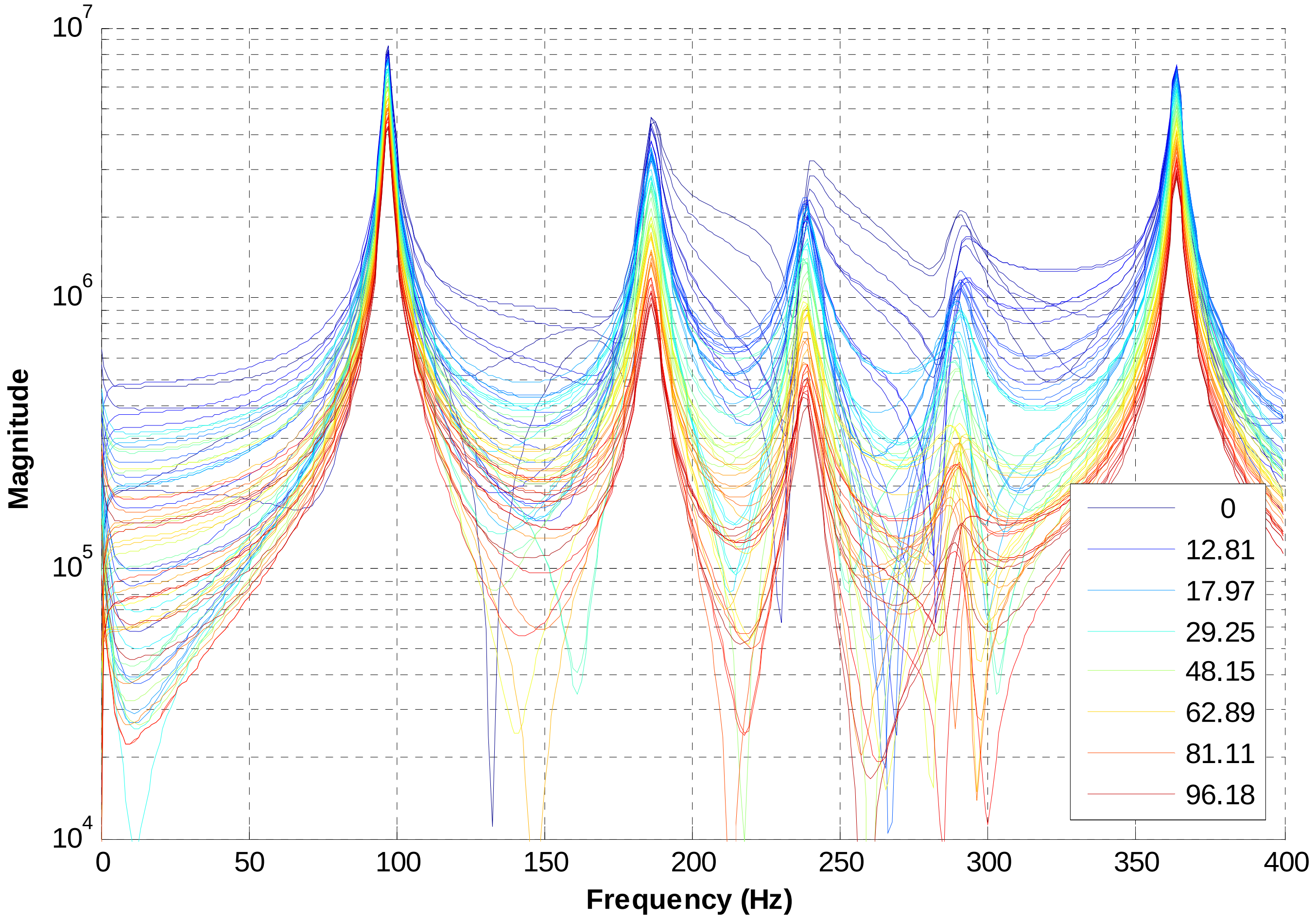
- 7-DOF system above used to test the methods.

Example #1 – Slip: Time Response



- Zero crossings in time response identified.
- It is difficult to discern if the system is nonlinear by simply inspecting the acceleration time history.

Example #1 – Slip: ZEFFTs



- ZEFFTs show extra frequency content between 180 and 350 Hz for zero times less than 15 ms.

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Truncated Analytical Impulse Response

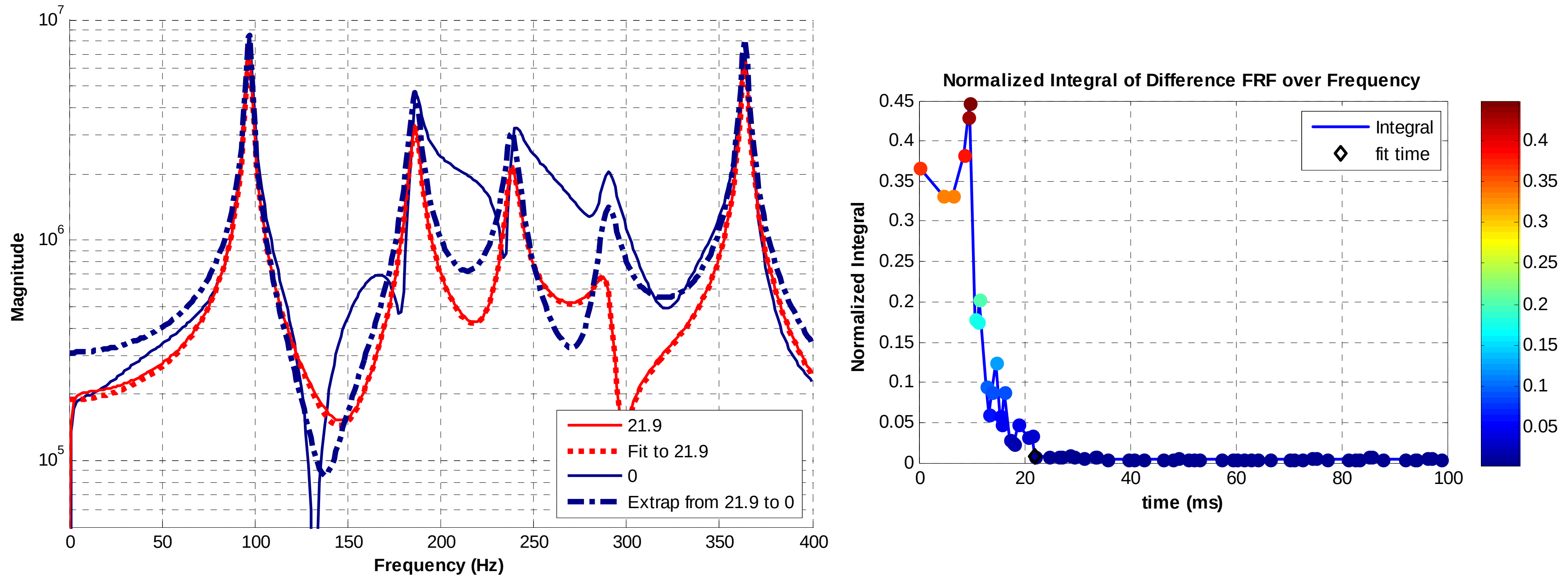
- Free response of an LTI system in Frequency Domain:

$$H(\omega) = \boxed{\sum_{r=1}^{2N} \frac{A_r}{i\omega - \lambda_r}} = \sum_{r=1}^{N} \left(\frac{A_r}{i\omega - \lambda_r} + \frac{A_r^*}{i\omega - \lambda_r^*} \right) = \sum_{r=1}^{N} \left(\frac{i\omega B_1 + B_0}{\omega_r^2 - \omega^2 + 2i\zeta_r \omega_r \omega} \right)$$

$$H_{t_z}(\omega) = \boxed{\sum_{r=1}^{2N} \frac{A_r e^{\lambda_r t_z}}{i\omega - \lambda_r} e^{i\omega t_z}}$$

Free response truncated at time t_z has the same form as the impulse response except for the $e^{i\omega t}$ factor.

Example #1 – Slip: BEND & IBEND



- Response at 21.9 ms was curve fit using AMI algorithm. Agreement is excellent at 21.9 ms and at all later times.
- Backwards extrapolation of 21.9 ms response to time 0 does not agree well suggesting that the system behaves nonlinearly some time before 21.9 ms.
- IBEND suggests that the system is linear for t > 10-15ms.

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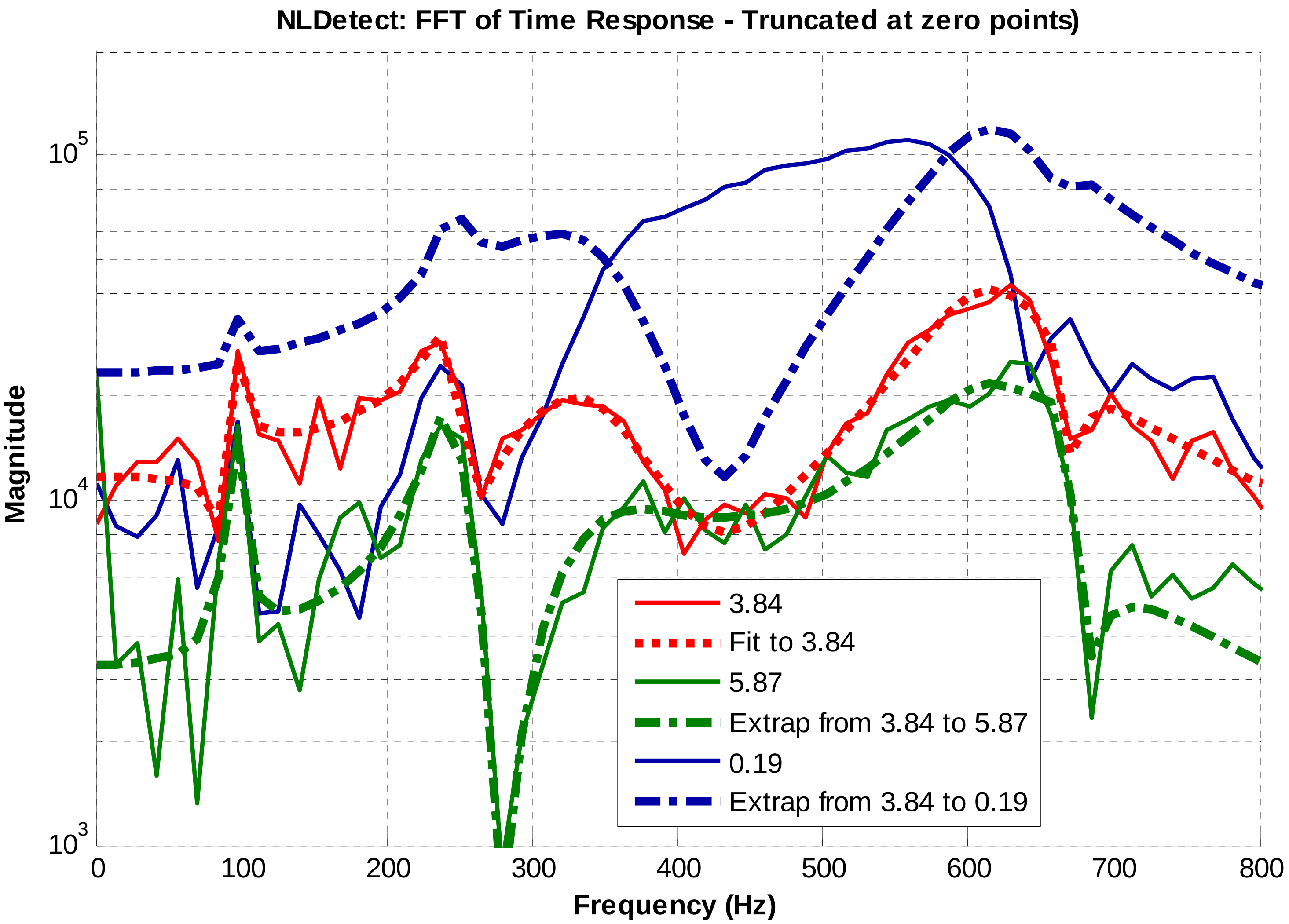
Shock Data: BEND



- Fit the response at 3.84ms and extrapolated backwards to 0.19ms and forwards to 5.87ms.
- Forward extrapolation agrees with the data about as well as the fit suggesting that the curve fit is accurate.
- Backward extrapolation does not show the broad peak between 375 and 600 Hz, suggesting that this is indeed due to nonlinearity.

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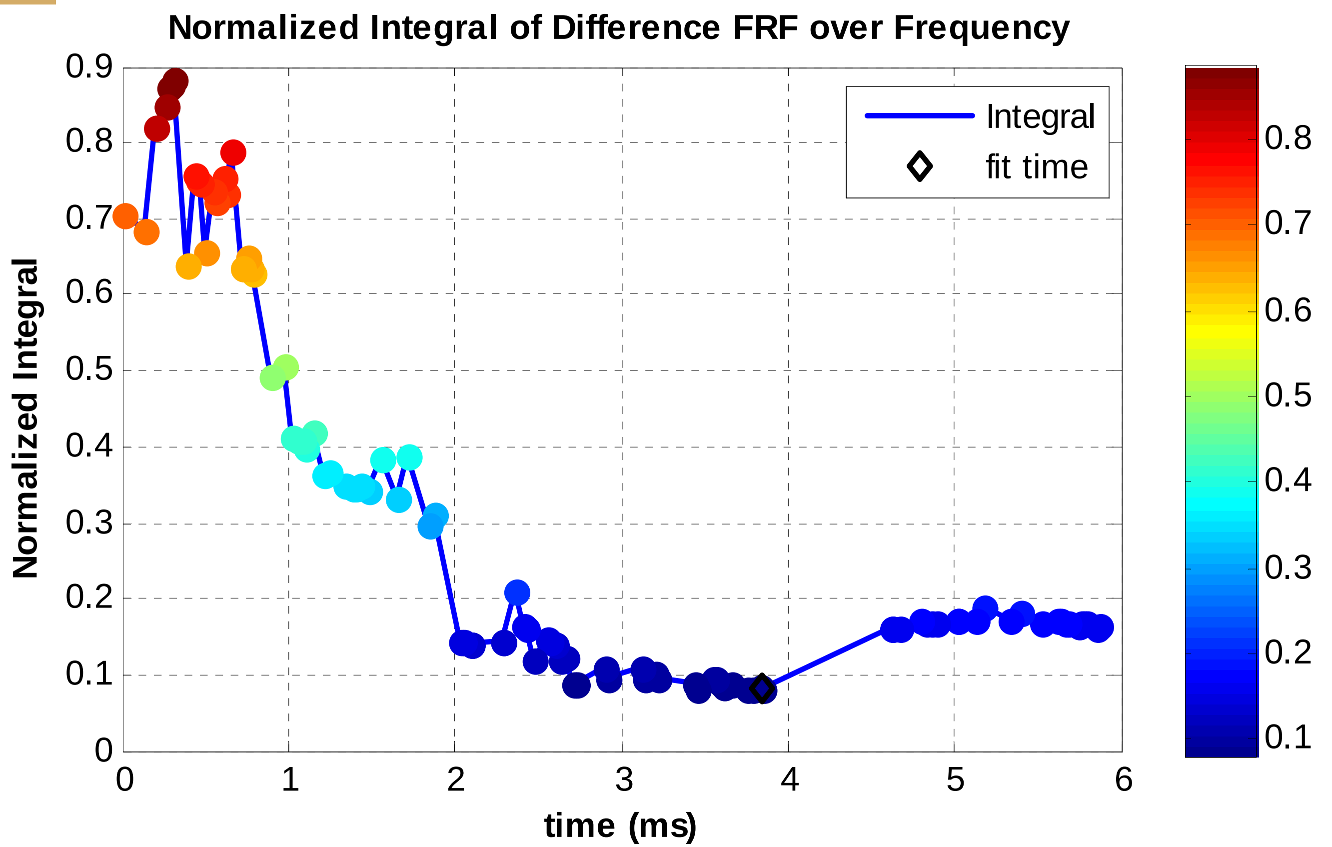
Shock Data: IBEND



- IBEND suggests that the system behaves linearly after 2ms.
- The agreement is not perfect from 4-6ms.
- This may be due to the increasing relative importance of measurement noise.

Conclusions

- Zeroed Early-time FFTs (ZEFFTs) and Backwards Extrapolation (BEND) provide insight into the response of a nonlinear system to shock loading.
- BEND and IBEND can be used to provide quantitative information and to develop insight.
  - Care must be taken when interpreting the results of linear system identification.
  - Even if SYSID fails, direct inspection of the ZEFFTs may still yield useful information.