MODEL UNCERTAINTY

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PLAN

* Uncertainty: parameter uncertainty - model uncertainty

* Benefits of Reduced Order Models

* Entropy Maximization

* Model Identification within an Uncertain Framework

PARAMETER UNCERTAINTY - MODEL UNCERTAINTY

= two sources of uncertainty preventing a perfect match of clean experimental measurements (no measurements uncertainty) with computational predictions:

Uncertainty on the parameters of the computational Model:

* variations (from part to part) of the values entered in the computational model, e.g. Young's modulus, Poisson's ratio, density, dimensions, etc.

* Test article and computational model are different parts from the same stock

PARAMETER UNCERTAINTY - MODEL UNCERTAINTY

Uncertainty (lack of "realism") of the computational model:

* computational geometry *approximates* certain features of the physical model, e.g.

  _ fasteners (rivets, welds, bolts, lap joints,…),

  _ plate/beam models of slender components, …

  _ no warping, out of straight,…

* boundary conditions typically differ from those in test/structure

* constitutive behavior is *modeled*: use of linear structural damping isotropic or orthotropic properties, linear elasticity,…

PARAMETER UNCERTAINTY - MODEL UNCERTAINTY

Notes:

* Separation between model and parameter uncertainty may be gray
  _ thickness can be varied as a parameter in plate/beam models but not in 3D blocks
  _ real boundary conditions can be approximated by linear springs (whose stiffness can become parameter uncertainty)
  _ meshless methods may help in making geometry uncertainty become parameter uncertainty.

* Model uncertainty cannot be eliminated, it may be reduced by great increases in parameter uncertainty.

REPRESENTATION/HANDLING OF UNCERTAINTY

"Throw randomness into the computations"

Not that easy if one desires to be physical/representative:

*Data Uncertainty (easiest): represent each of the uncertainty data as a random variable (if fixed within the element, random process or field otherwise).

The complete representation of a set of *n* random variables requires the specification of the *joint* probability density function, which is a function of *n* variables and includes variation levels (standard deviations) and "correlation/dependence" type information. Is such a complete data available? (*No*) Does it matter?

REPRESENTATION/HANDLING OF UNCERTAINTY

*Model Uncertainty: one could run a *few* different models. Is that sufficient? (*most likely not*). Maybe one can introduce randomness somewhere as in data uncertainty…?

*Model Updating of Uncertain Structures:

_ Accuracy of "mean (computational) model" is not primary focus, rather it is accuracy of the predicted band of uncertainty around the mean model predictions.

_ Mean model updating need to be carried out to capture the physics that affects the band of uncertainty

BENEFITS OF REDUCED ORDER MODELS

*Reduced Order Models: *when successful/appropriate*

_ reduce the complexity of the uncertainty modeling problem, i.e. number of uncertain parameters

_ transform *model* uncertainty into *parameter* uncertainty

_ should be carried out with *fixed basis* appropriately determined

_ reduce computational cost of carrying out Monte Carlo simulations to assess uncertainty effects

_ eliminate *topological* model constraints (e.g. banded stiffness matrix)

_ reduce/eliminate obvious correlation between various uncertain parameters of the model

ENTROPY MAXIMIZATION

What do we do if we don't have (as is usual) the complete model of all random variables describing data uncertainty in the ROM?

One answer: (postulating the specific model is another)
Derive the necessary model from an engineering vision of /desire for the uncertainty.

One such vision/desire is that the uncertainty is not simply limited to a small neighborhood of the mean model but spreads broadly as allowed given a set of *constraints*.

This approach leads to the
constrained maximization of the statistical entropy
for the determination of the probabilistic model of uncertainty

EXAMPLE

Linear structural dynamic reduced order model involves mass, damping, and stiffness matrices with following properties:

These matrices are: (i) symmetric, and (ii) positive definite.

The first problem of maximization of entropy was thus about simulating random matrices with properties (i) and (ii) and
(iii) mean of random matrices = matrices of "mean model"
(iv) no zero eigenvalue should occur in the random matrices if none exists in the mean model.

Solution of this problem by Christian Soize in 2000.

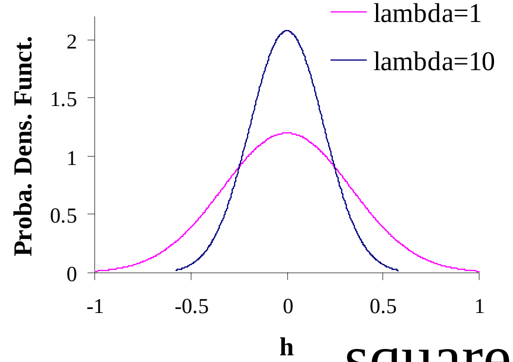
SOLUTION

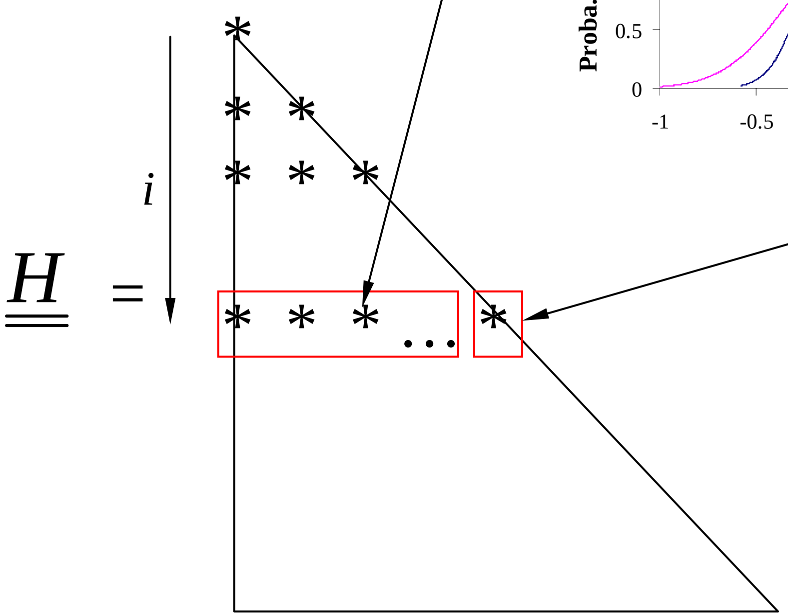
$$\underline{\underline{\bar{A}}} = \underline{\underline{\bar{L}}}\,\underline{\underline{\bar{L}}}^T$$

$$\underline{\underline{A}} = \underline{\underline{\bar{L}}}\,\underline{\underline{G}}\,\underline{\underline{\bar{L}}}^T$$

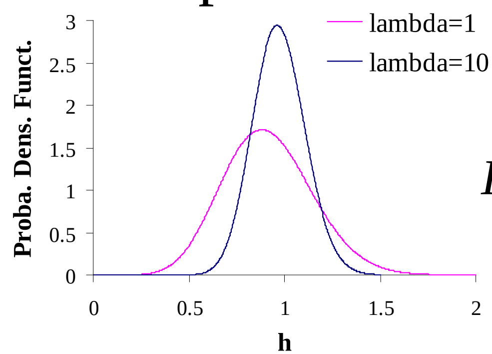
$$\underline{\underline{G}} = \underline{\underline{H}}\,\underline{\underline{H}}^T$$

zero mean Gaussian, independent of each other with standard dev. $\sigma_{il} = 1/\sqrt{2\mu}$





square root of Gamma, independent of all others



$$H_{ii} = \sqrt{\frac{Y_{ii}}{\mu}}$$

EXTENSIONS

* problems with rigid body modes
* matrices that are not symmetric, positive definite:
  acoustic-structure interface, bearing stiffness/damping matrices,…
* uncertainty on linear boundary/attachment conditions
* more information on the level of uncertainty (variance of nat. freq.)
* nonlinear geometric ROMs, i.e. linear and nonlinear stiffness terms
* coupled matrices, e.g. mass - gyroscopic matrices in rotordynamics
* …

Entropy Maximization can also serve as basis for simulation of random processes (e.g., friction coefficients), and fields (e.g., random elasticity tensor)

MODEL IDENTIFICATION IN UNCERTAIN FRAMEWORK

The uncertainty model obtained from entropy maximization has parameters =

(a) parameters of mean model (e.g. natural frequencies, damping ratios)

(b) parameters describing the uncertainty level (Lagrange multipliers associated with the constraints).

These uncertainty model parameters can be obtained using classical estimation approaches (e.g., maximum likelihood) to provide an updating of the mean model from an ensemble of measurements on random parts.