Shakedown at frictional contact

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- A. Klarbring, M. Ciavarella, J.R. Barber, Shakedown in elastic contact problems with Coulomb friction, International Journal of Solids and Structures, Volume 44, Issues 25-26, 15 December 2007, Pages 8355-8365
- James R. Barber, Anders Klarbring, Michele Ciavarella, Shakedown in frictional contact problems for the continuum, Comptes Rendus Mécanique, Volume 336, Issues 1-2, January-February 2008, Pages 34-41
- Young Ju Ahn, Enrico Bertocchi, J.R. Barber, Shakedown of coupled two-dimensional discrete frictional systems, Journal of the Mechanics and Physics of Solids, Volume 56, Issue 12, December 2008, Pages 3433-3440

We consider a system of contacting elastic bodies, discretized using the finite element method with incremental Coulomb friction boundary conditions.

The external loading

$$\boldsymbol{F}(t) = \boldsymbol{F}_0 + \lambda \boldsymbol{F}_1(t)$$

comprises a constant 'mean load' $\boldsymbol{F}_0$ and a periodic time-varying load $\lambda \boldsymbol{F}_1(t)$ where λ is a scalar load factor.

We assume that the loading is never sufficient to cause 'gross slip' (sliding) in which all the contact nodes slip at the same time. However, at any given time a subset of the contact nodes may slip - a state known as 'microslip'.

After an initial transient, the long-time steady state might involve:-

- *Shakedown*: Slip displacements during the transient phase lead to a state of residual stress that prevents all slip in the steady state.
- *Cyclic slip*: The slip scenario is exactly repeated during each loading period and the total accumulated slip at each node during one period is zero.
- *Ratcheting*: The slip scenario is exactly repeated during each loading period, but a constant total slip is accumulated at each node during one period each period. This is possible only if the system has a rigid-body mode.

This behaviour is closely analogous with that of elastic-plastic systems under periodic loading.

However, there are important differences:-

1) The 'failure' (i.e. slip) condition for Coulomb friction is a dimensionless coefficient of friction, rather than a yield stress.
One consequence is that if the complete loading scenario $\boldsymbol{F}(t)$ is multiplied by a positive scalar factor, the long term behaviour will be qualitatively unchanged and the nodal forces and displacements will all be increased by the same factor.

2) The flow rule for frictional slip is non-associative.

For elastic-plastic deformation with an associative flow rule, Melan's theorem applies.

Many frictional systems appear to obey a form of Melan's theorem.

However, the proof of Melan's theorem for plasticity depends on the flow rule being associative and this is not satisfied by the Coulomb friction law.

We have recently proved that Melan's theorem applies to elastic systems with Coulomb friction contact conditions if and only if there is no coupling between tangential (slip) displacements and normal contact reactions.

Counter examples can be found for all coupled systems in that the final steady state can depend upon the initial conditions.

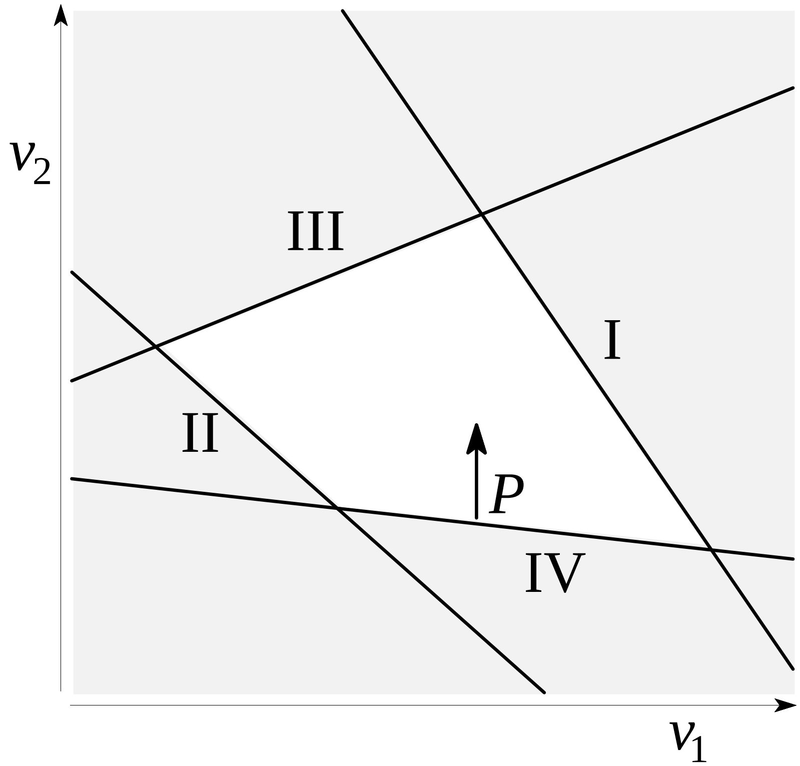
We assume all nodes remain in contact $(w_i = 0)$ and define the instantaneous state as a point P in the space v_i of slip displacements.

The frictional constraints (incipient forward or backward slip) at node j are defined by

$$
\begin{aligned}
(A_{ji} - fB_{ji})v_i &\leq fp_j^w - q_j^w \\
(A_{ji} + fB_{ji})v_i &\geq -fp_j^w - q_j^w \ ,
\end{aligned}
$$

Each of these $2N$ constraints (2 for each node) is a directional hyperplane in v_i-space.

We illustrate this for a simple two-node system.



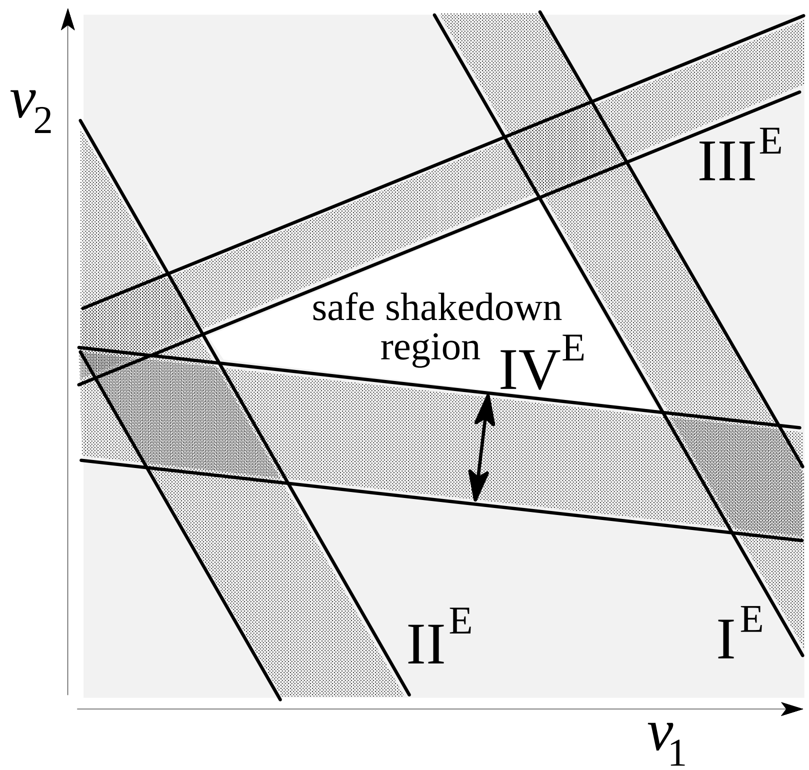
The four constraints exclude the (shaded) region of v_1v_2-space on one side of the lines I,II,III,IV respectively.

The instatansous state of the system must lie in the unshaded region.

If the external loads cause IV to move to exclude more space, P is 'pushed' upwards ($\dot{v}_2 > 0$).

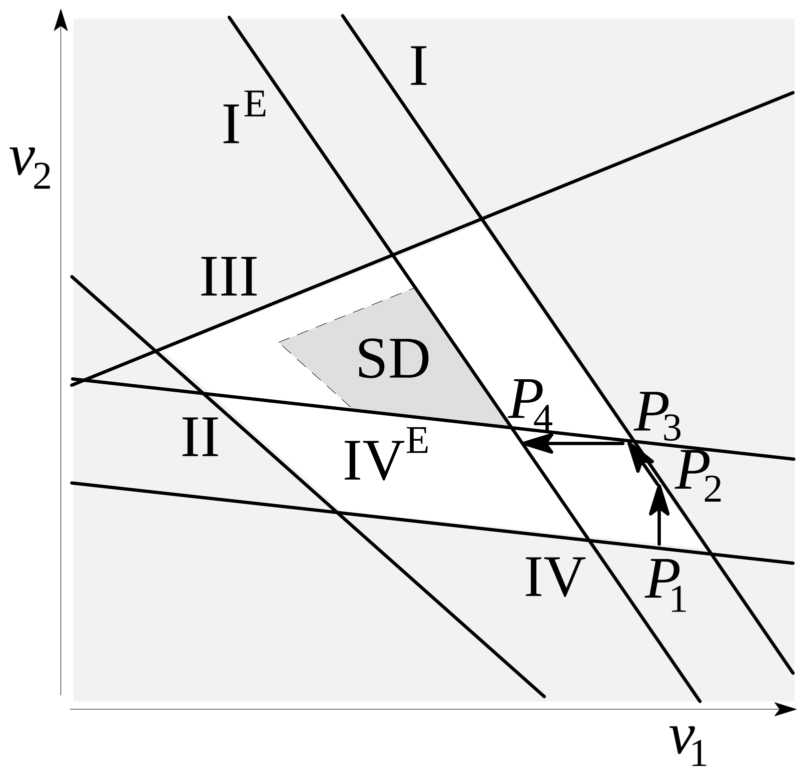
Now imagine the external loads changing periodically, so that the lines I,II,III,IV move back and forth as shown.

We can identify the extreme positions I^E etc. where each constraint excludes the maximum space.



Shakedown is possible if and only if these extreme constraints leave a non-null safe shakedown region.

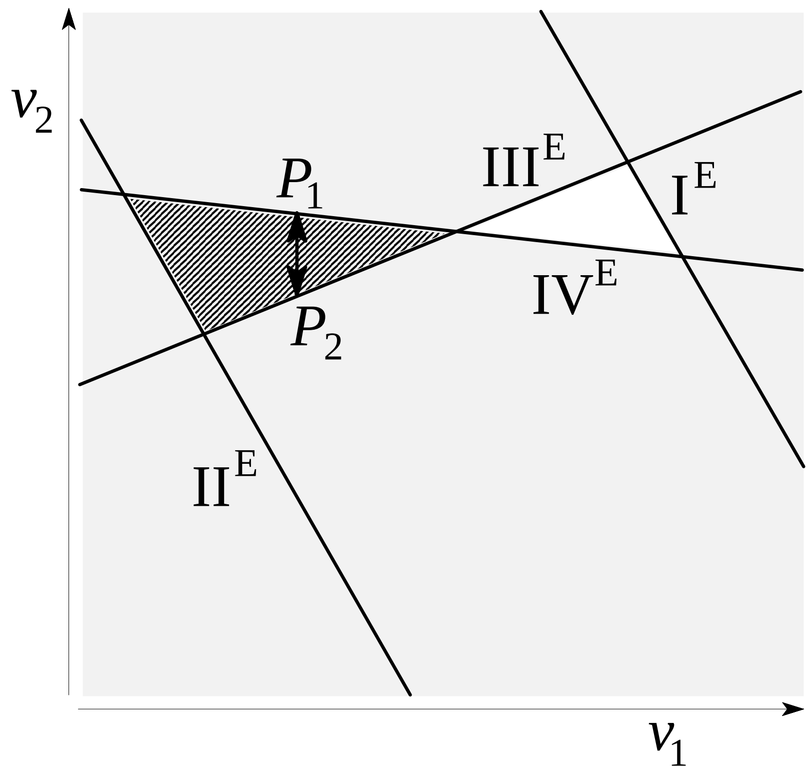
Suppose IV advances to IV^E and then recedes, after which I advances to I^E and recedes.



The point P is moved monotonically towards the safe shakedown region SD.

This is always true if the safe shakedown region is a quadrilateral.

If the safe shakedown region is a triangle, the steady state can be either cyclic slip or shakedown, depending on the initial condition (position of P).



In the case illustrated, P could end up cycling between P_1 and P_2.

If the scalar load factor λ is increased from zero, the safe shakedown region is initially quadrilateral. At some critical value λ_L it becomes triangular, and then at a higher critical value λ_U it becomes null.

- For $\lambda < \lambda_L$ we have shakedown for all initial conditions.
- For $\lambda_L < \lambda < \lambda_U$ we may have shakedown or cyclic slip depending on the initial conditions.
- For $\lambda > \lambda_U$ shakedown is impossible and we have cyclic slip for all initial conditions.

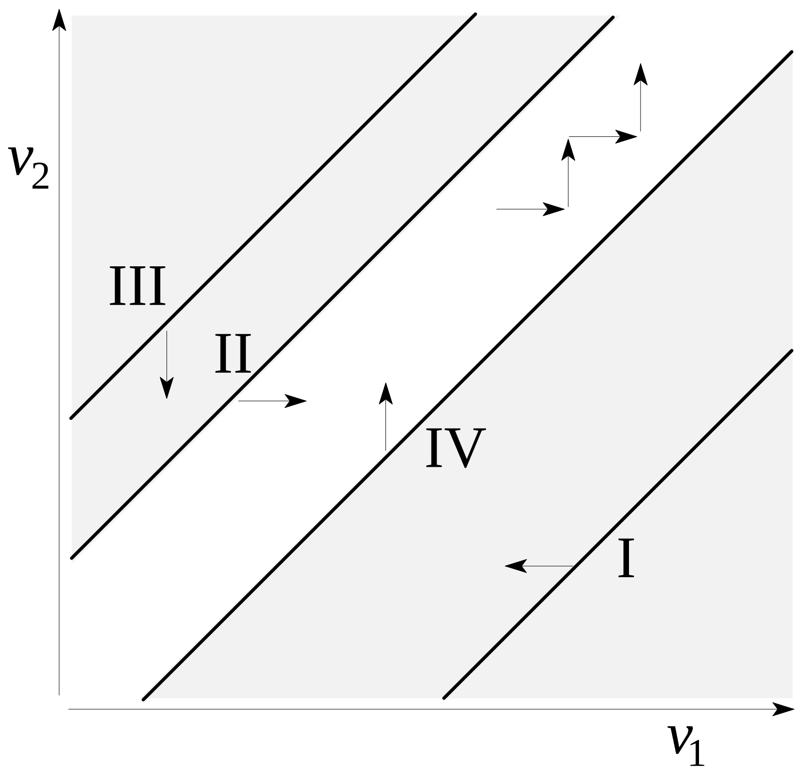
If the system is uncoupled, the constraints are parallel in pairs and the safe region is a parallelogram. It can therefore never reduce to a triangle and $\lambda_L = \lambda_U$.

This confirms that Melan's theorem applies when the system is uncoupled.

Ratcheting

If the system has a rigid-body mode, the stiffness matrix is singular and the constraints are all parallel to a common line.

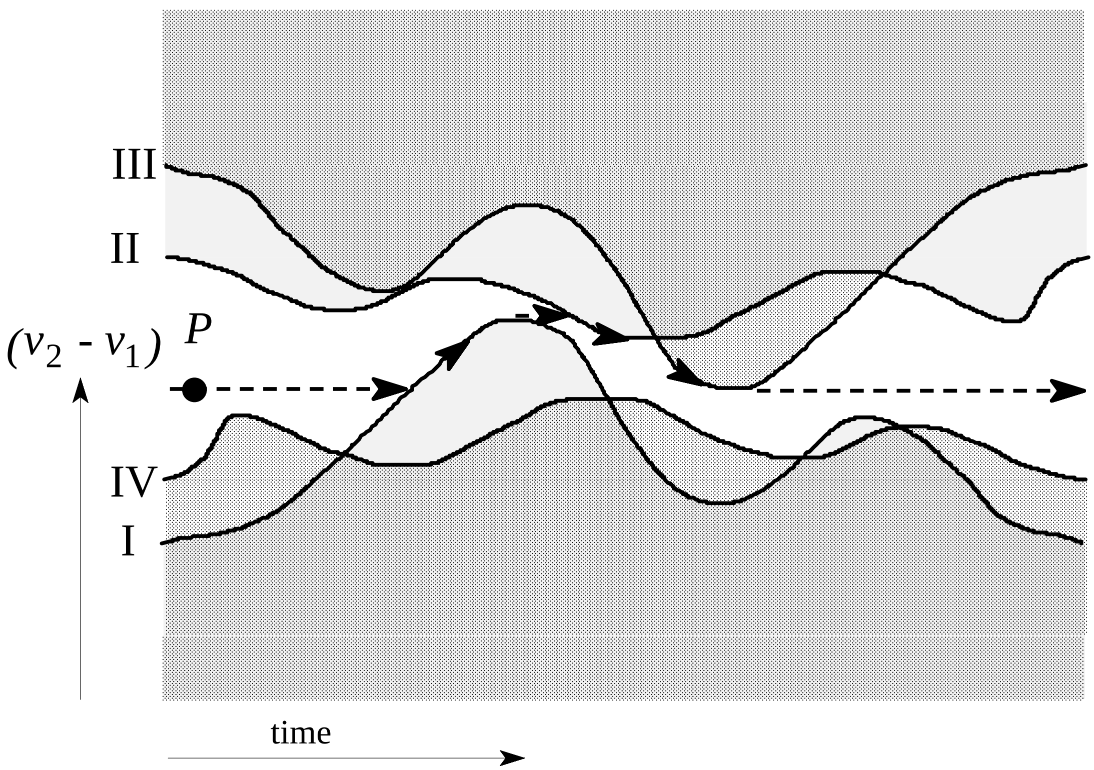
For the two-node system with possible rigid-body motion $v_1 = v_2$, they are at 45^o.



With appropriate motion of the constraints, we can obtain the ratcheting motion illustrated.

However, if there is a safe shakedown region (a strip in this case), the system must shakedown.

Motion of P depends on which of the constraints is most advanced at a given time. We can project these positions onto the line orthogonal to the rigid-body motion and plot them as a function of time.



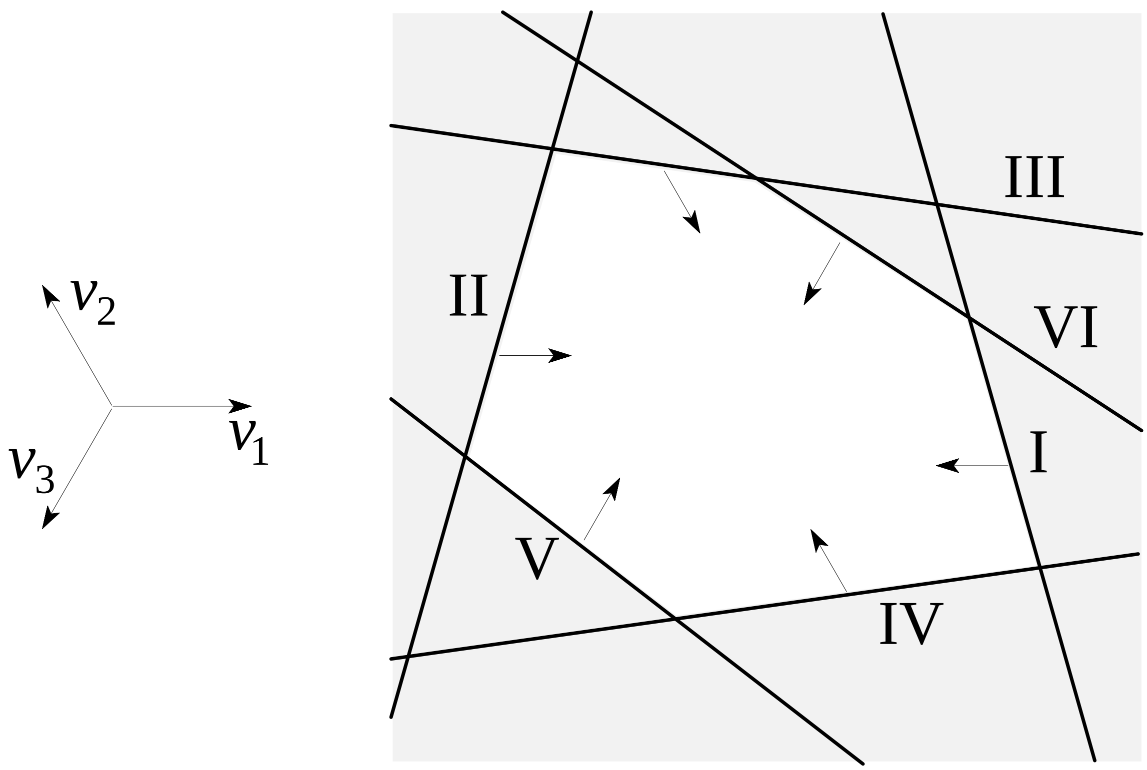
The motion of P is equivalent to the trajectory of a ball dropped through the space between the constraints in the figure.

Either cyclic slip or ratcheting is possible depending on the sequence of constraint motions.

Depending on the exact form of the periodic load $\boldsymbol{F}_1(t)$, increasing λ can cause a transition to either ratcheting or cyclic slip and further transitions between these states can occur at higher values of λ.

For a three-node system with a rigid-body mode, the constraints will be planes all parallel to a common line $v_1 = v_2 = v_3$.

Looking along this line, we can track the deformation of the structure in response to the (now six) moving constraints.



Conclusions

The motion of the point $P(v_1, v_2, ...)$ in v_i-space provides useful information about the kinds of behaviour to be expected under time-varying loads.

The lower bound λ_L below which shakedown occurs for all initial conditions can be found by solving subsets of the frictional constraints as equalities and checking the remaining constraints as inequalities.

The upper bound λ_U above which shakedown is impossible can also be found this way, or alternatively by using a constrained optimization technique.

In systems with a rigid body mode, all the constraints are parallel to a given line and evolution of the system can be tracked by projecting the space onto a hyperplane perpendicular to this line.

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