UNCERTAINTY MODELING ISSUES: SOME PRELIMINARIES

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EPISTEMIC VS. ALEATORY

Epistemic (or model or reducible) uncertainty

Observed when the response of the system cannot be matched by the model predictions irrespectively of the model parameters, e.g. curved beam modeled by a straight one, nonlinear system represented by a linear one, …

Aleatory (or parameter or irreducible) uncertainty

Observed when the response of the system can be matched by the model predictions for an appropriate choice of the parameters which is different for different structures, e.g. random Young' modulus

Improving the model tends to reduce epistemic uncertainty but increase aleatory uncertainty

EPISTEMIC VS. ALEATORY

The “type” of model affects the balance of epistemic/aleatory uncertainty

Detailed (finite element) model:

Aleatory uncertainty can be introduced only in the “mechanical/ material” properties

Global (modal) model:

Aleatory uncertainty extends to broad set of parameters (e.g. elements of stiffness matrix) that can include some uncertainty seen as epistemic in the detailed model, e.g. curvature of beam including in stiffness matrix

UNCERTAINTY MODELING: NOT AN AFTERTHOUGHT!

Scenario 1: All structures of interest are tested
Proceed with deterministic identification and adjust model parameters from structure to structure. *No uncertainty modeling is needed.*

Scenario 2: a few nominally identical structures are tested and many more are of interest (usual case). *Uncertainty modeling is needed*. *How?*

Example: FRF of a *N* dof system is measured and damping ratios ζ_i are observed to be uncertain.

How do we proceed? Two options…

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(1) proceed with deterministic identification and backtrack an uncertain model of the parameters.

$FRF^{(j)} \rightarrow \underline{\zeta}^{(j)}$ using deterministic ID, then represent $\underline{\zeta}^{(j)}$ using an uncertainty model and identify the parameters $\underline{\theta}$ of this model.

(2) create and identify an uncertain model of the structure (or model of the uncertain structure) that combines structural and uncertainty aspects. That is, express

$$FRF^{(j)} = FRF^{(j)}\left[\underline{\zeta}^{(j)}\left(\underline{\theta}\right)\right]$$

and identify directly (e.g. maximum likelihood) the parameters $\underline{\theta}$.

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UNCERTAIN MODEL VALIDATION

Uncertain model = structural + uncertainty model

Option 1:
Validate the structural model in detail (epistemic uncertainty?) and the uncertainty modeling separately.

Option 2:
Validate the overall model on the responses of interest and assess whether the model *statistically* predicts these responses, e.g. they lie within the 5-95 percentile confidence (uncertainty) band with 10% probability.

Should we still focus on validating/improving structural model?

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MODELING COMPLEXITY

Does the model used when uncertainty is present need to exhibit full complexity?

Not necessarily - maybe or maybe not…

Fine details in the response may not need to be captured as they will become “invisible” when uncertainty is introduced.

<u>Classical Example:</u>
Probability density function (stationary) of the response of a Duffing oscillator to white noise excitation does not require the classic single-frequency analyses but its spectrum estimation may involve it.

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Tools for Uncertainty Modeling:
Probability Theory, Fuzzy Logic, Possibility Theory,...

Random variables, stochastic processes and fields:
Uncertain parameters modeled through their *joint* probability density function the estimation of which in general requires an extraordinary amount of information

Assumptions are necessary!

(a) Ad-hoc distribution selection
(b) “Stochastic Parametrizations”
(c) Maximum entropy approach

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(a) Ad-hoc distribution modeling

Use a combination of independence assumptions (joints $\rightarrow$ marginals) and prescribed distributions (Gaussian/normal, lognormal, uniform,..) to characterize the problem. Issues:

* many different "types" of assumptions
* danger of violating physics

Example: Gaussian distribution for stiffness is often accepted if the mean/standard deviation is large (say 10) as probability of negative value is "small". Yet, mathematically the variance of response is ∞.

This issue is reflected by the non-convergence of the sample variance as the number of samples increases.

One solution: truncated Gaussian but where to truncate?

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(b) "Stochastic Parametrization"

Represent random variables (processes, etc.) in a "modal" form, i.e. through an expansion on a random basis but with deterministic parameters. Most notable: polynomial chaos (PC) representation, e.g. for a single random variable

$$P = \sum_{l=0} \gamma_l \, Q_l(V)$$

* V is a random variable with a specified distribution
* Q_l are specified functions (orthogonal polynomials)
* γ_l are deterministic parameters characterizing the random variable P

Still at risk of violating physics unless implemented in V, Q_l, and γ_l .

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(c) Maximum Entropy Approach

The joint probability density needed is not chosen, it is *derived* to maximize the statistical entropy

$$S = -\int_{\Omega} p_{\underline{\underline{A}}}\left(\underline{\underline{a}}\right) \ln\, p_{\underline{\underline{A}}}\left(\underline{\underline{a}}\right) d\,\underline{\underline{a}}$$

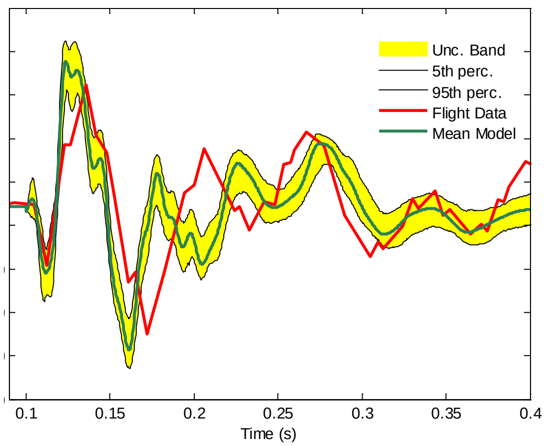
subject to a series of physical and data matching constraints.

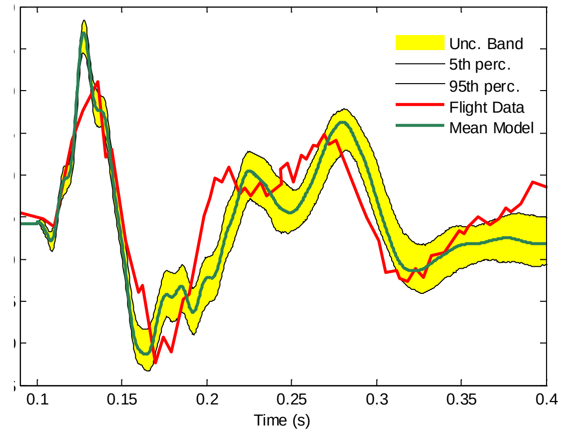
physical constraints: matrix symmetry, positive property, boundedness reflected in Ω.

data matching constraints: mean, standard deviation, ….

The Lagrange multipliers associated with the data matching constraints become the parameters of the distribution.

AN EXAMPLE





Aircraft response during missile launch