Identification of Nonlinear Bolted Lap-Joint Parameters using Force-State Mapping

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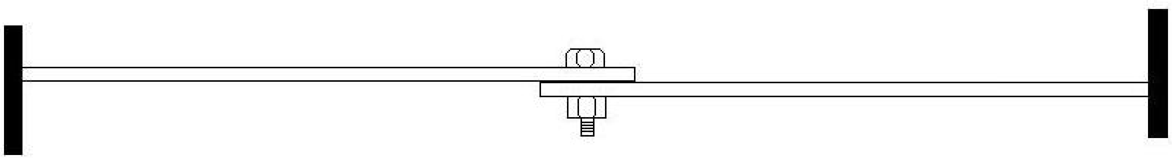

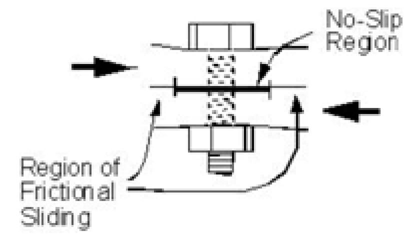

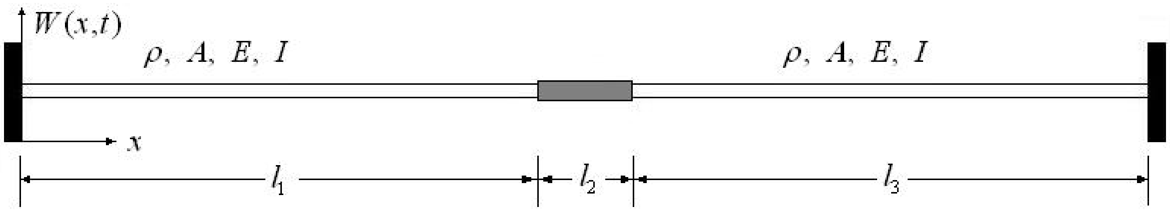

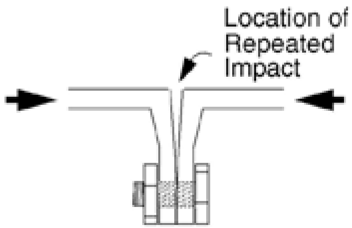

$$\bar{\Gamma}\bar{d} = \bar{F} \qquad \bar{\Gamma} = \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma_J \end{bmatrix}$$

$$\boxed{\Gamma_J = \Gamma_L + \Gamma_{NL}} \Rightarrow \boxed{\Gamma_{NL} = h(x, \dot{x})}$$

- Joint modeling means identification of the joint operator both qualitatively and quantitatively.

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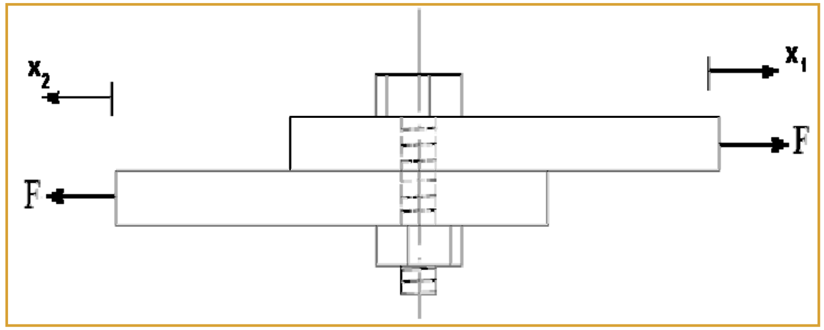

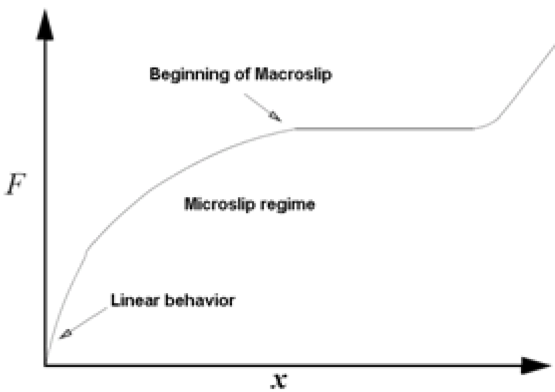

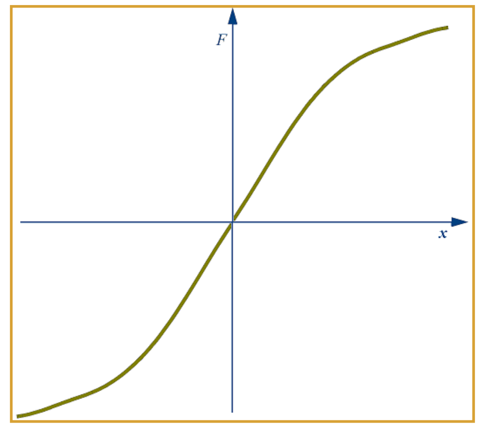

- Static tangential loading leads to a softening stiffness effect:

$$F(x) = k_1 x + k_3 x^3 + k_5 x^5 + \ldots$$

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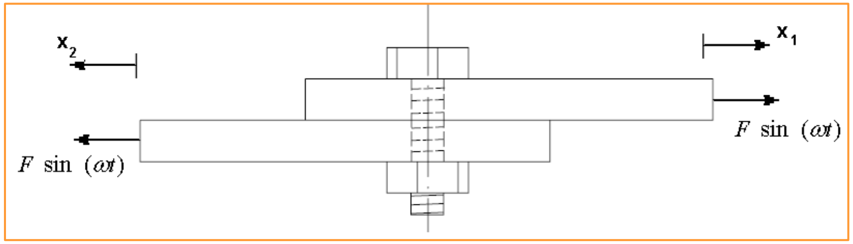

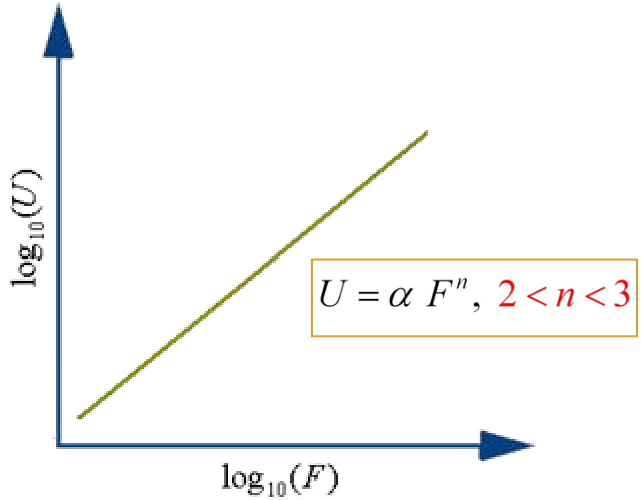

- Harmonic force at the bolted lap joint
- Energy dissipated vs. the force amplitude follows a power-law relationship.
- Experimental results show that $2 \leq n \leq 3$
- Linear viscous damping – dependent upon maximum steady-state displacement amplitude

$$c(x_{max}) = c_0 + c_1 x_{max} + c_2 x_{max}^2 + \cdots$$

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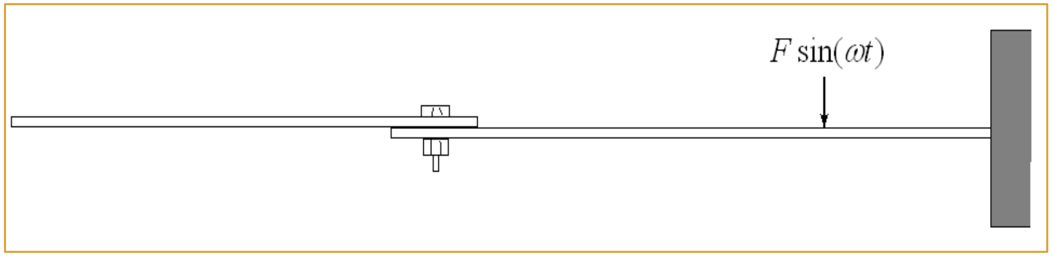
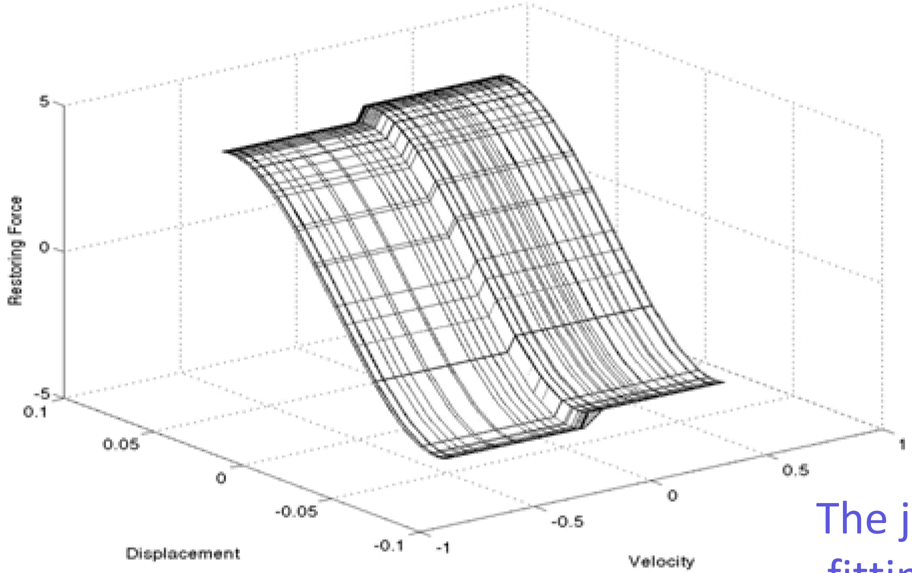
Force-State Mapping





Equations in modal coordinates:

$$\ddot{q}(t) + \omega_1^2 q(t) + h(\dot{q}, q) = Q(t)$$

$$\omega_1^2 q(t) + h(q, \dot{q}) = Q(t) - \ddot{q}(t)$$

The joint parameters are identified by fitting a function on the surface .

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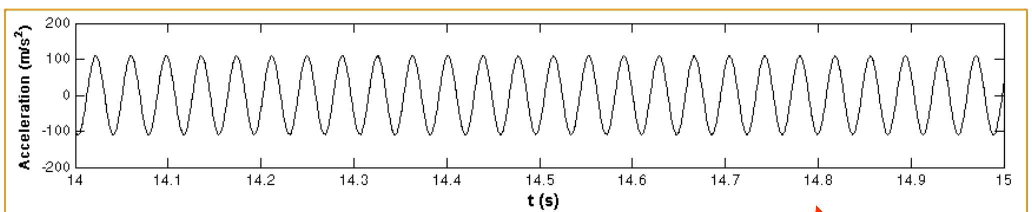
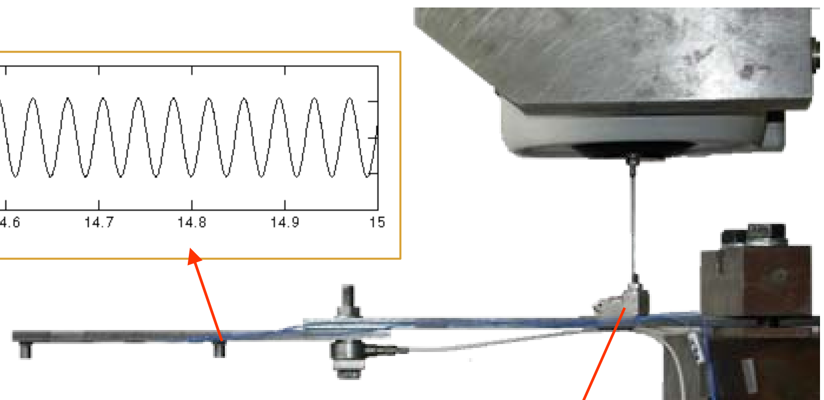
Experiment

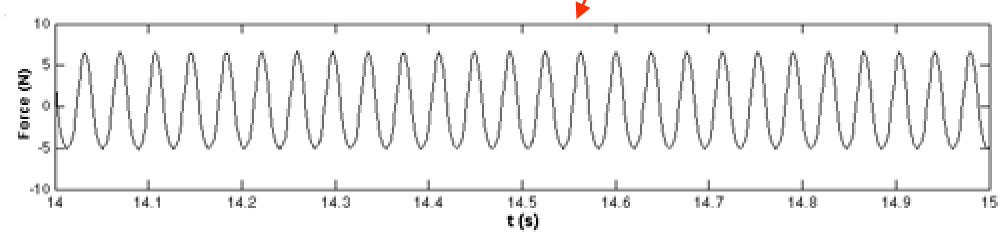
- Single-frequency excitation close to a natural frequency
- Measured low-level force and acceleration converted to a single modal coordinate using an updated FE model
- Other (higher) modes are eliminated
- Modal acceleration integrated analytically to obtain velocity and displacement

$$\ddot{q}(t) = \sum_{i}^{n} \left(A_i \sin(i\omega t) + B_i \cos(i\omega t) \right)$$

- Truncated after the fourth harmonic







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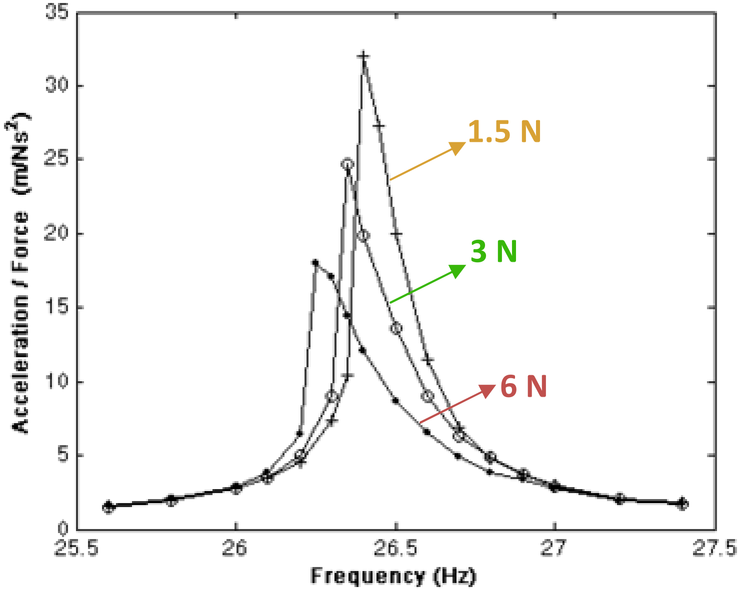
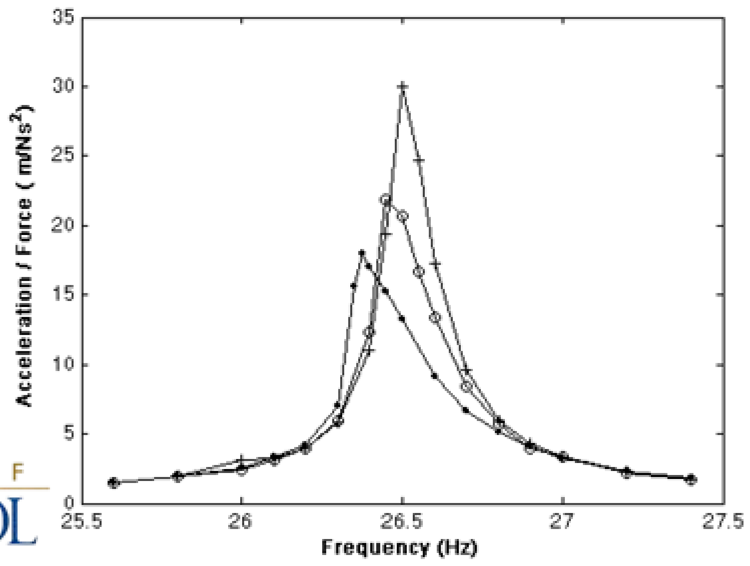
Preload=120N



3 Excitation levels

2 Preload conditions

Preload=540N



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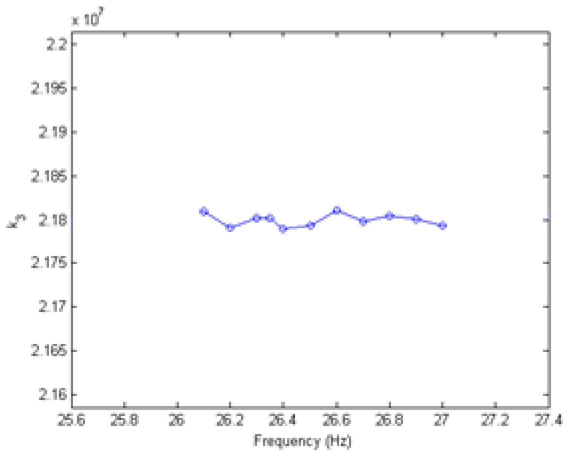

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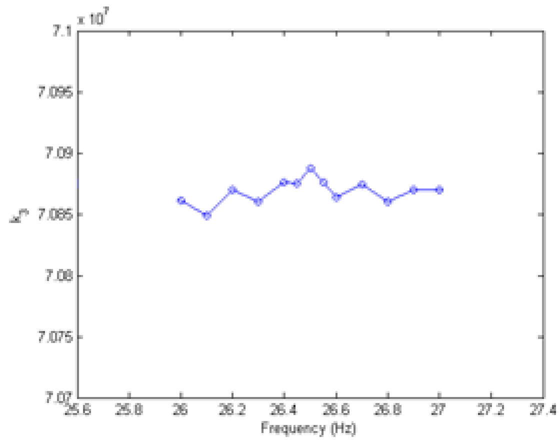

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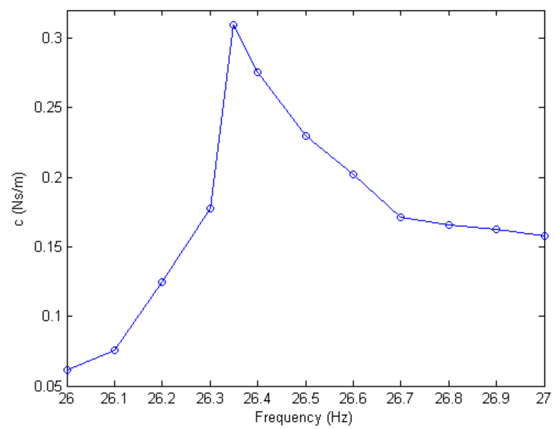
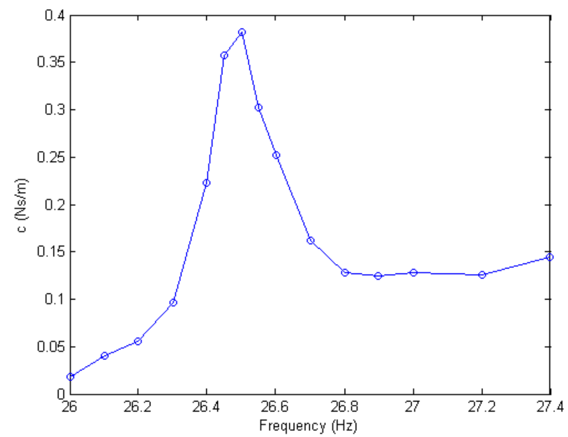
Identification shows that k_3 is almost constant





Viscous damping *c* peaks at the natural frequency – it depends upon the amplitude of vibration

$$Q(t) - \ddot{q}(t) = \omega_1^2 q(t) - k_3 q(t)^3 + c(q_{max})\dot{q}(t)$$

$$c(q_{\max}) = c_0 + c_1 q_{\max} + c_2 q_{\max}^2 + \cdots$$

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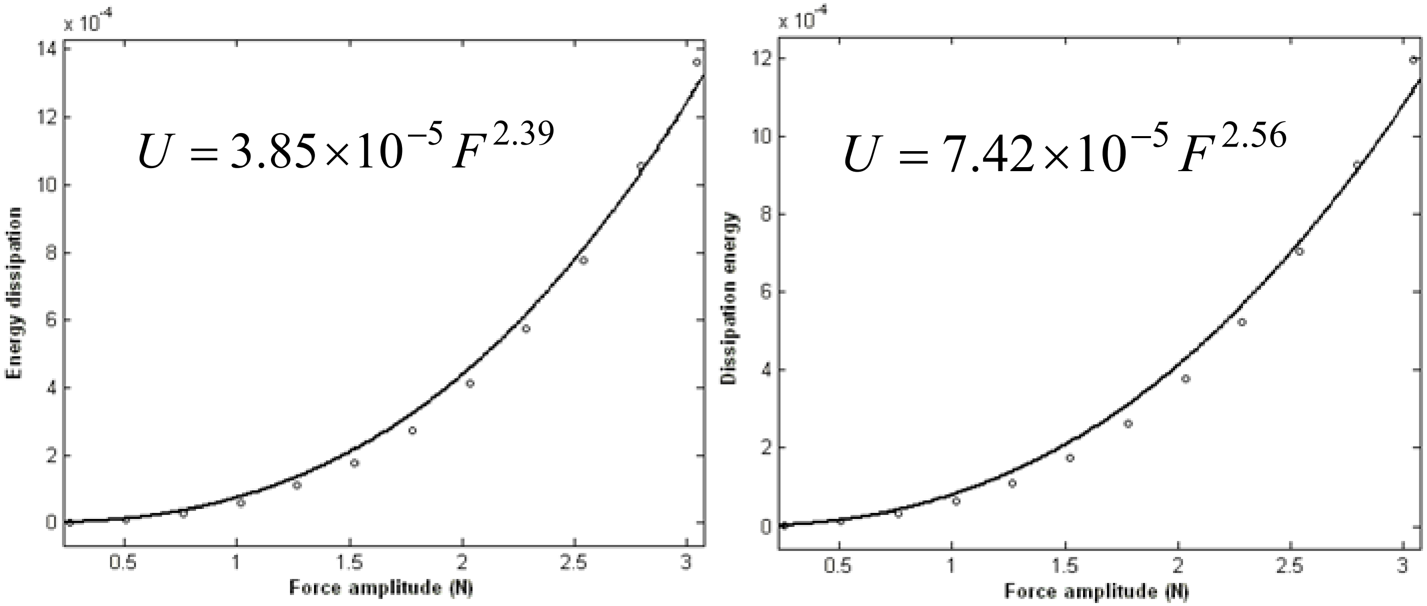
Displacement – dependent damping

$$c(q_{max}) = c_0 + c_1 q_{max} + c_2 q_{max}^2 + \cdots$$

Energy constraint

$$U = \alpha\, F^n,\ 2 < n < 3$$



Preload 120N

Preload 540N

Excitation at 6N

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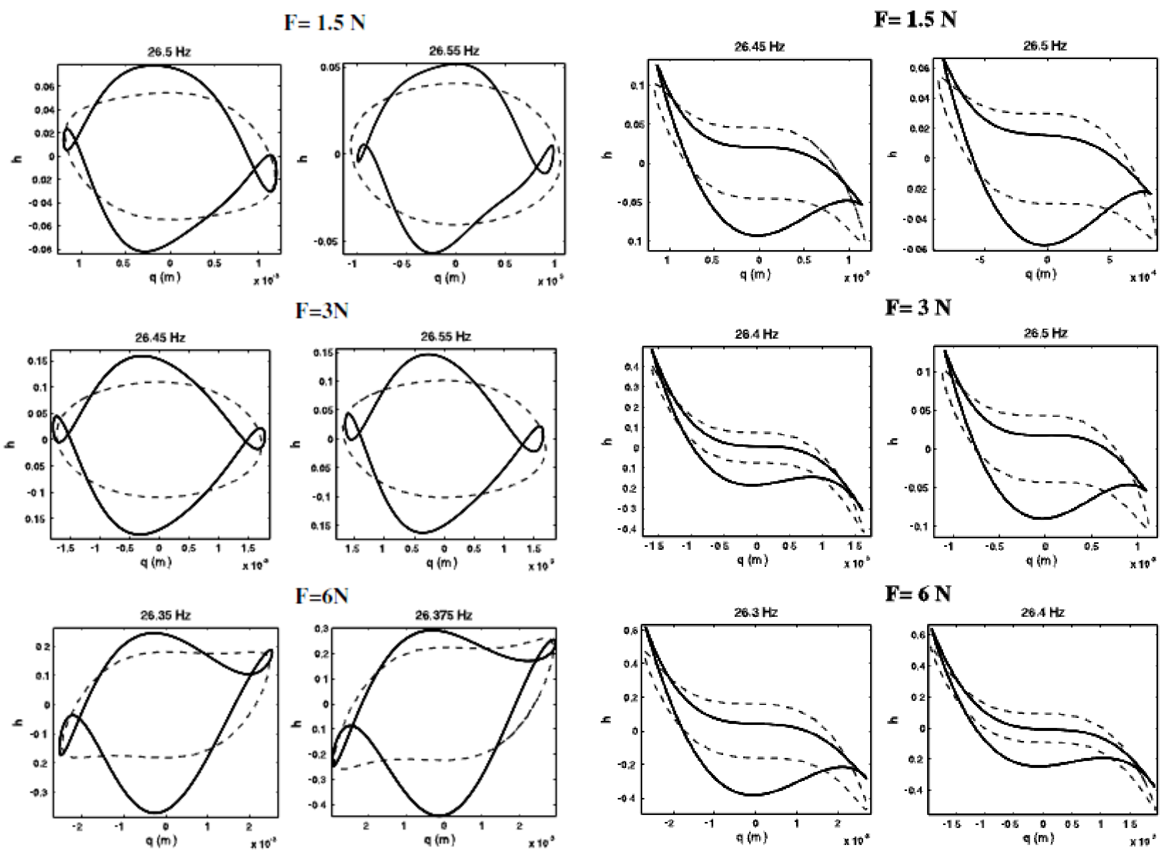

Full line – experimental
Dashed line- analytical

Hysteresis Loops – Preload=120N

- Area and orientation approximately correct
- Possible underestimation of the cubic stiffening term

Perturbation Methods for the Estimation of Parameter Variability in Stochastic Model Updating

Mechanical Systems and Signal Processing 22 (2008) 1751-1773
Hamed Haddad Khodaparast, John E Mottershead and Michael I Friswell

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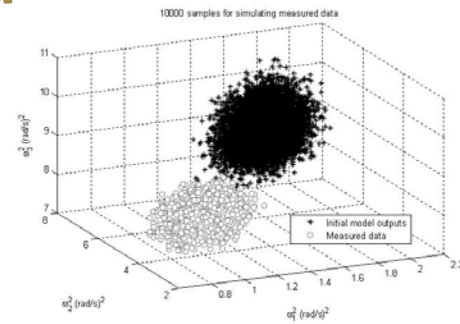
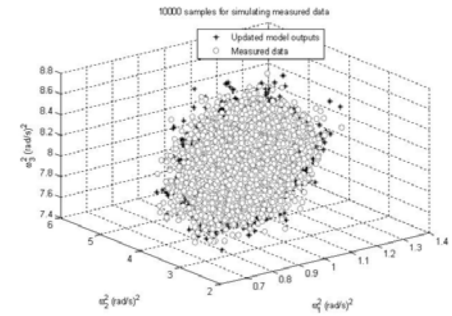
The Perturbation Method





Classical model updating: $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \mathbf{T}_j(\mathbf{z}_m - \mathbf{z}_j)$

Measurement: $\mathbf{z}_m = \overline{\mathbf{z}}_m + \Delta\mathbf{z}_m$

Prediction: $\mathbf{z}_j = \overline{\mathbf{z}}_j + \Delta\mathbf{z}_j$

Parameters: $\boldsymbol{\theta} = \overline{\boldsymbol{\theta}} + \Delta\boldsymbol{\theta}$

Transformation matrix: $\mathbf{T} = \overline{\mathbf{T}} + \Delta\mathbf{T}_j$

$$\Delta\mathbf{T}_j = \sum_{k=1}^{n} \frac{\partial \mathbf{T}_j}{\partial z_{mk}} \Delta z_{mk}$$

Mean $\overline{\bullet}$
Variability Δ

Stochastic model updating:

$$\overline{\boldsymbol{\theta}}_{j+1} + \Delta\boldsymbol{\theta}_{j+1} = \overline{\boldsymbol{\theta}}_j + \Delta\boldsymbol{\theta}_j + \left(\overline{\mathbf{T}}_j + \Delta\mathbf{T}_j\right)\left(\overline{\mathbf{z}}_m + \Delta\mathbf{z}_m - \overline{\mathbf{z}}_j - \Delta\mathbf{z}_j\right)$$

The Perturbation Method

$\mathbf{O}\left(\Delta^0\right)$: $$\bar{\boldsymbol{\theta}}_{j+1} = \bar{\boldsymbol{\theta}}_j + \bar{\mathbf{T}}_j\left(\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j\right)$$

- the mean values of the updating parameters

$\mathbf{O}(\Delta^1)$: $$\Delta\boldsymbol{\theta}_{j+1} = \Delta\boldsymbol{\theta}_j + \bar{\mathbf{T}}_j\left(\Delta\mathbf{z}_m - \Delta\mathbf{z}_j\right) + \left(\left(\sum_{k=1}^{n}\frac{\partial\bar{\mathbf{T}}_j}{\partial z_{mk}}\Delta z_{mk}\right)\left(\bar{\mathbf{z}}_m - \bar{\mathbf{z}}_j\right)\right)$$

- leads to an expression for the parameter covariances

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Parameter Covariances

$$\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_{j+1},\boldsymbol{\Delta\theta}_{j+1}\right)=\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j+\mathbf{A}_j\,\boldsymbol{\Delta}\mathbf{z}_m+\overline{\mathbf{T}}_j\left(\boldsymbol{\Delta}\mathbf{z}_m-\boldsymbol{\Delta}\mathbf{z}_j\right),\;\boldsymbol{\Delta\theta}_j+\mathbf{A}_j\,\boldsymbol{\Delta}\mathbf{z}_m+\overline{\mathbf{T}}_j\left(\boldsymbol{\Delta}\mathbf{z}_m-\boldsymbol{\Delta}\mathbf{z}_j\right)\right)$$

$$\begin{aligned}
&=\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta\theta}_j\right)+\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta}\mathbf{z}_m\right)\mathbf{A}_j^T+\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta}\mathbf{z}_m\right)\overline{\mathbf{T}}_j^T-\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T\\
&\quad+\left(\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta}\mathbf{z}_m\right)\mathbf{A}_j^T\right)^T+\mathbf{A}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_m\right)\mathbf{A}_j^T+\mathbf{A}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_m\right)\overline{\mathbf{T}}_j^T-\mathbf{A}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T\\
&\quad+\left(\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta}\mathbf{z}_m\right)\overline{\mathbf{T}}_j^T\right)^T+\left(\mathbf{A}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_m\right)\overline{\mathbf{T}}_j^T\right)^T+\overline{\mathbf{T}}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_m\right)\overline{\mathbf{T}}_j^T-\overline{\mathbf{T}}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T\\
&\quad-\left(\mathrm{Cov}\left(\boldsymbol{\Delta\theta}_j,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T\right)^T-\left(\mathbf{A}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T\right)^T-\left(\overline{\mathbf{T}}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_m,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T\right)^T+\overline{\mathbf{T}}_j\mathrm{Cov}\left(\boldsymbol{\Delta}\mathbf{z}_j,\boldsymbol{\Delta}\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T
\end{aligned}$$

$$\mathbf{A}=\left[\frac{\partial\overline{\mathbf{T}}_j}{\partial z_{m1}}\left(\overline{\mathbf{z}}_m-\overline{\mathbf{z}}_j\right)\;\;\frac{\partial\overline{\mathbf{T}}_j}{\partial z_{m2}}\left(\overline{\mathbf{z}}_m-\overline{\mathbf{z}}_j\right)\;\cdots\;\frac{\partial\overline{\mathbf{T}}_j}{\partial z_{mn}}\left(\overline{\mathbf{z}}_m-\overline{\mathbf{z}}_j\right)\right]\qquad\overline{\mathbf{T}}_j=\left(\overline{\mathbf{S}}_j^T\mathbf{W}_1\overline{\mathbf{S}}_j+\mathbf{W}_2\right)^{-1}\overline{\mathbf{S}}_j^T\mathbf{W}_1$$

$\mathrm{Cov}(\Delta\theta_j,\Delta z_j)$and $\mathrm{Cov}(\Delta z_j,\Delta z_j)$ are determined by forward propagation

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$$\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j+1},\Delta\mathbf{z}_{\mathrm{m}}\right)=\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j}+\mathbf{A}_{j}\Delta\mathbf{z}_{m}+\overline{\mathbf{T}}_{\mathrm{j}}\left(\Delta\mathbf{z}_{m}-\Delta\mathbf{z}_{j}\right),\Delta\mathbf{z}_{m}\right)$$

$$=\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{m}\right)+\left(\mathbf{A}_{j}+\overline{\mathbf{T}}_{j}\right)\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{m}\right)-\overline{\mathbf{T}}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{j},\Delta\mathbf{z}_{m}\right)$$

$$\mathrm{Cov}\left(\Delta\mathbf{z}_{j},\Delta\mathbf{z}_{m}\right)=\overline{\mathbf{S}}_{\mathrm{j}}\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{m}\right)$$

$$\mathbf{A}_{j+1}=\left[\left.\frac{\partial\overline{\mathbf{T}}_{j+1}}{\partial z_{m1}}\right|_{z_{m1}=\bar{z}_{m1}}\left(\overline{\mathbf{z}}_{m}-\overline{\mathbf{z}}_{j+1}\right)\quad \left.\frac{\partial\overline{\mathbf{T}}_{j+1}}{\partial z_{m2}}\right|_{z_{m2}=\bar{z}_{m2}}\left(\overline{\mathbf{z}}_{m}-\overline{\mathbf{z}}_{j+1}\right)\quad\cdots\quad \left.\frac{\partial\overline{\mathbf{T}}_{j+1}}{\partial z_{mn}}\right|_{z_{mn}=\bar{z}_{mn}}\left(\overline{\mathbf{z}}_{m}-\overline{\mathbf{z}}_{j+1}\right)\right]$$

$$\left.\frac{\partial\overline{\mathbf{T}}_{j+1}}{\partial z_{mk}}\right|_{z_{mk}=\bar{z}_{mk}}=\sum_{i=1}^{m}\frac{\partial\overline{\mathbf{T}}_{j+1}}{\partial\bar{\theta}_{(j+1),i}}\left.\frac{\partial\bar{\theta}_{(j+1),i}}{\partial z_{mk}}\right|_{z_{mk}=\bar{z}_{mk}}\quad;\quad k=1,2,\ldots,n$$

$$\frac{\partial\overline{\mathbf{T}}_{j+1}}{\partial\theta_{(j+1),i}}=\left(\overline{\mathbf{S}}_{j+1}^{T}\mathbf{W}_{1}\overline{\mathbf{S}}_{\mathrm{j+1}}+\mathbf{W}_{2}\right)^{-1}\frac{\partial\overline{\mathbf{S}}_{j+1}^{T}}{\partial\theta_{(j+1),i}}\mathbf{W}_{1}$$

$$-\left(\overline{\mathbf{S}}_{j+1}^{T}\mathbf{W}_{1}\overline{\mathbf{S}}_{j+1}+\mathbf{W}_{2}\right)^{-1}\left(\frac{\partial\overline{\mathbf{S}}_{j+1}^{T}}{\partial\theta_{(j+1),i}}\mathbf{W}_{1}\overline{\mathbf{S}}_{j+1}+\overline{\mathbf{S}}_{j+1}^{T}\mathbf{W}_{1}\frac{\partial\overline{\mathbf{S}}_{j+1}}{\partial\bar{\theta}_{(j+1),i}}\right)\left(\overline{\mathbf{S}}_{j+1}^{T}\mathbf{W}_{1}\overline{\mathbf{S}}_{j+1}+\mathbf{W}_{2}\right)^{-1}\overline{\mathbf{S}}_{j+1}^{T}\mathbf{W}_{1}$$

2nd order sensitivity needed

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Simplification

If the measurements and parameters are assumed to be uncorrelated then,
$\mathrm{Cov}(\Delta\mathbf{z}_m,\Delta\boldsymbol{\theta}_j)=0$, $\mathrm{Cov}(\Delta\mathbf{z}_m, \Delta\mathbf{z}_j)=0$

The parameter covariances become;

$$\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j+1},\Delta\boldsymbol{\theta}_{j+1}\right)=\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_j,\Delta\boldsymbol{\theta}_j\right)-\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_j,\Delta\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T+\overline{\mathbf{T}}_j\mathrm{Cov}\left(\Delta\mathbf{z}_m,\Delta\mathbf{z}_m\right)\overline{\mathbf{T}}_j^T$$
$$-\overline{\mathbf{T}}_j\mathrm{Cov}\left(\Delta\mathbf{z}_j,\Delta\boldsymbol{\theta}_j\right)+\overline{\mathbf{T}}_j\mathrm{Cov}\left(\Delta\mathbf{z}_j,\Delta\mathbf{z}_j\right)\overline{\mathbf{T}}_j^T$$

No requirement for the second-order sensitivities.

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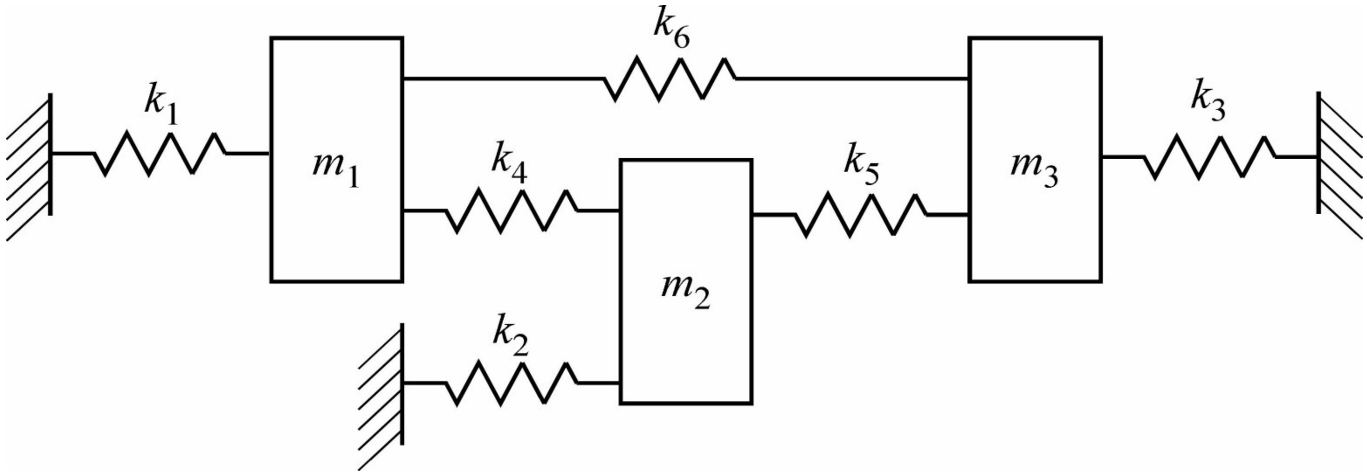

Known deterministic parameters

$m_i = 1.0\,\text{kg} \quad (\text{i} = 1,2,3) \qquad k_i = 1.0\,\text{N/m} \quad (i = 3,4) \qquad k_6 = 3.0 \;\text{N/m}$

Unknown Gaussian random variables with mean values and standard deviations given by

$\mu_{k_1} = 1.0\,\text{N/m} \quad , \mu_{k_2} = 1.0\,\text{N/m}\,, \; \mu_{k_5} = 1.0\,\text{N/m}$

$\sigma_{k1} = 0.20\text{N/m}, \quad \sigma_{k2} = 0.20\text{N/m}, \quad \sigma_{k5} = 0.20\text{N/m}$

The measured data are obtained by using Monte Carlo simulation.
The initial estimates of the unknown random parameters are,

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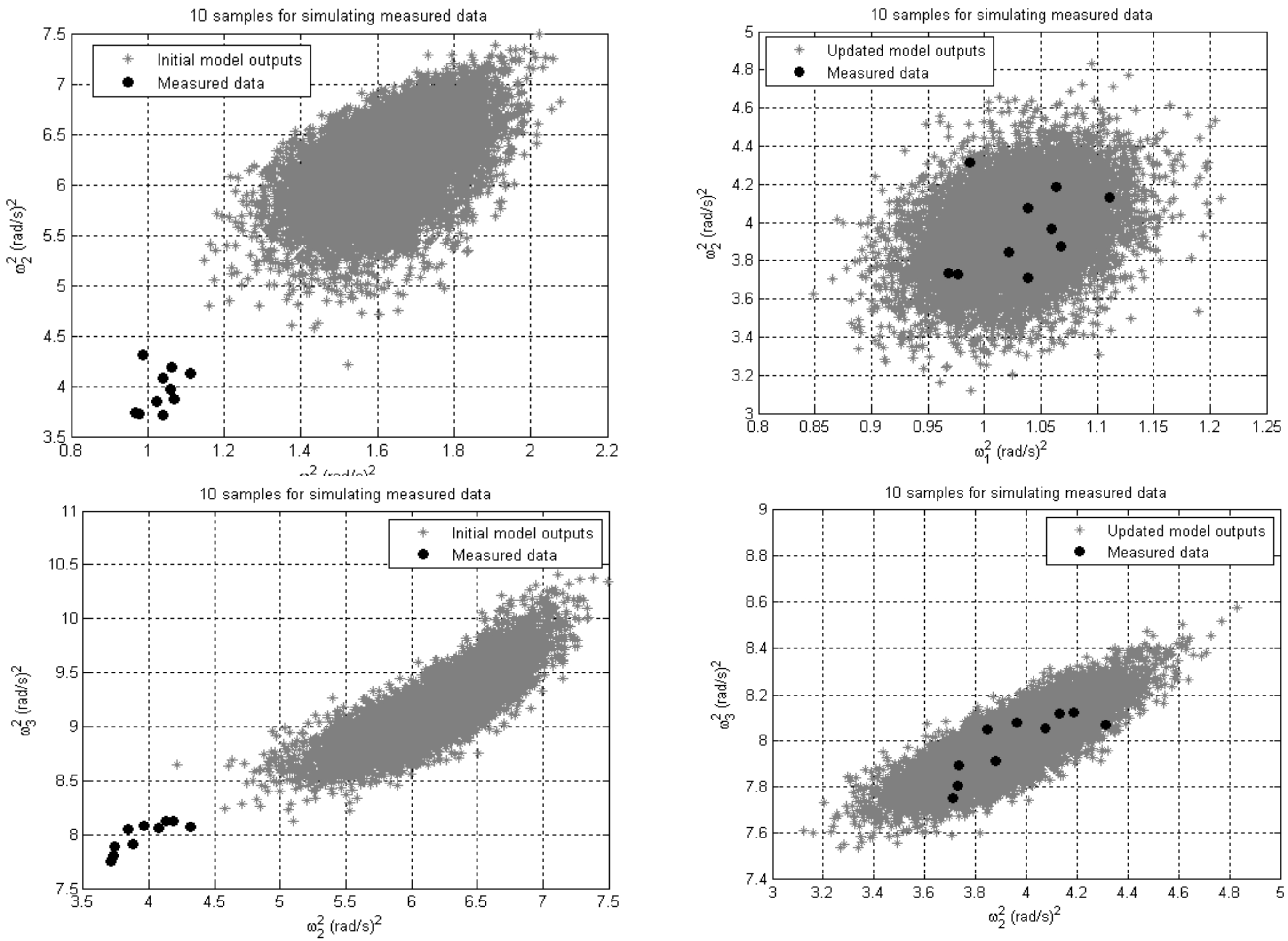
$\bar{k}_1 = \bar{k}_2 = \bar{k}_5 = 2.0\,\text{N/m} \quad \text{Cov}(k_i) = (0.3^2)\text{N}^2/\text{m}^2 \;\; i = 1,2,5$



Convergence of natural frequency distributions

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Converged Distributions – 10000 Samples

Parameters	Initial % Error	% Error (1)	% Error (2)	% Error (3)	% Error (4)	% Error (5)
$\bar{k}_1$	100	1.29	1.43	1.22	1.88	16.40
$\bar{k}_2$	100	-2.45	-2.70	-2.68	-2.62	36.56
$\bar{k}_5$	100	0.56	0.61	0.61	2.13	58.58
$std\left(\bar{k}_1\right)$	50	0.14	0.45	1.50	-89.88	-14.82
$std\left(\bar{k}_2\right)$	50	-0.68	1.96	0.75	-89.96	-13.07
$std\left(\bar{k}_5\right)$	50	1.22	0.62	0.34	-90.07	-59.87

Methods:
1. Simplified perturbation method
2. Full perturbation method
3. Perturbation method by Hua
4. Minimum variance – Collins et al.
5. Minimum variance - Friswell

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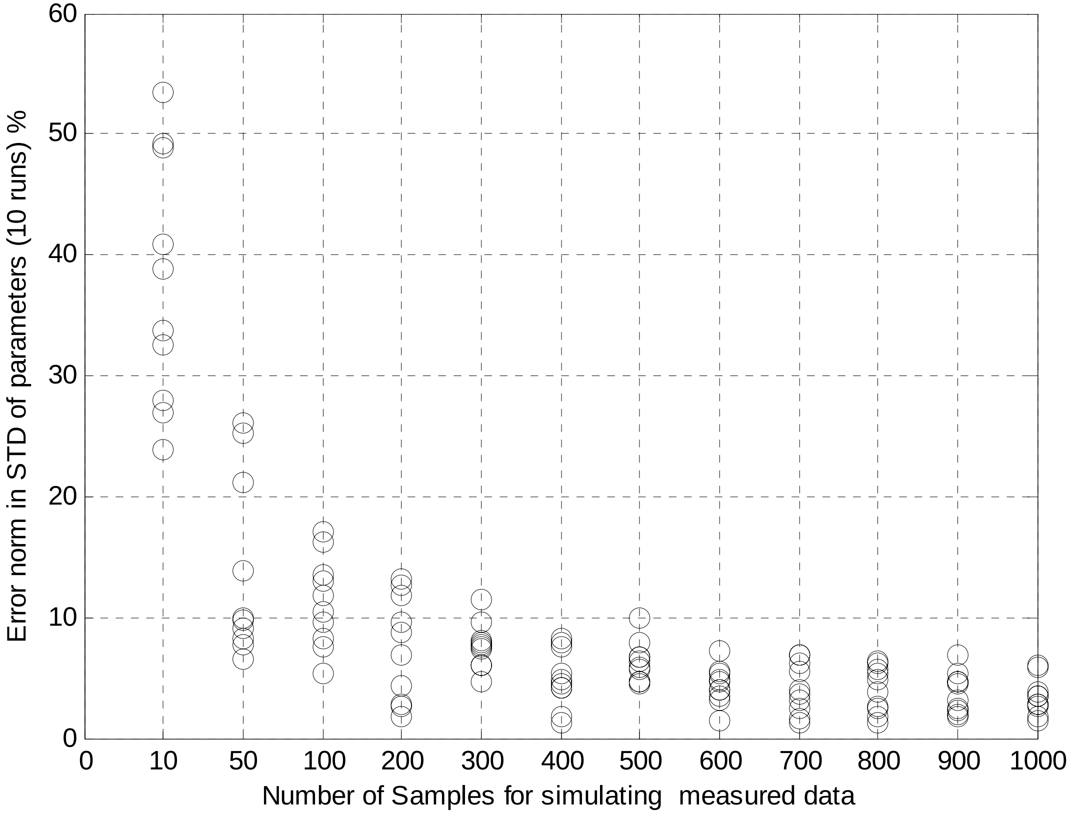
Effect of sample size



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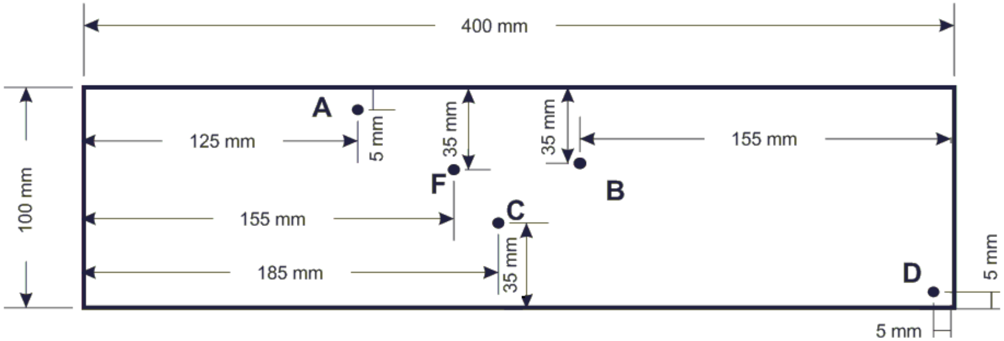
Experimental Case Study – Plate Thickness Variability

Arrangement of accelerometers (A, B, C, D) and driving point (F)



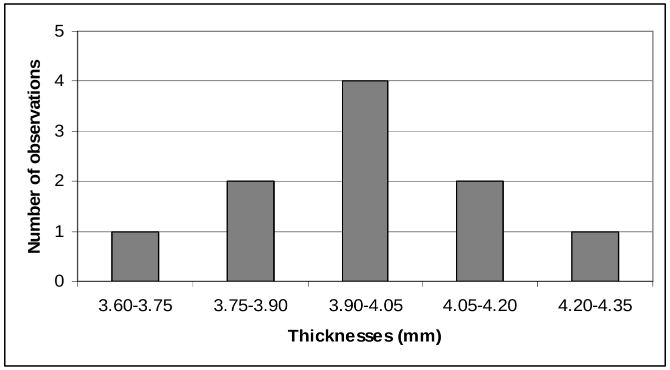


Plate thickness distribution

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Measurement Distribution from 10 plates

	Mode Number				
Plate No.	1	2	3	4	5
1	119.774	284.283	331.970	589.404	656.359
2	121.615	291.922	337.186	605.160	665.854
3	123.156	291.440	340.184	602.603	673.357
4	128.048	298.163	355.210	620.139	700.798
5	128.533	303.809	357.110	630.809	704.505
6	128.596	301.010	361.488	635.533	713.207
7	129.796	311.726	361.114	646.765	712.792
8	135.058	315.393	374.368	653.584	738.395
9	134.478	312.215	374.406	649.130	737.256
10	138.141	321.812	382.932	667.203	755.189
Mean	128.720	303.177	357.597	630.033	705.771
STD	6.011	12.032	17.048	25.235	32.854

	Mode Number				
Plate No.	6	7	8	9	10
1	932.576	1091.603	1343.097	1628.879	1825.215
2	953.666	1106.861	1372.890	1650.395	1860.225
3	955.515	1119.445	1376.298	1669.899	1868.071
4	980.403	1165.177	1414.181	1736.714	1924.260
5	995.188	1169.660	1433.020	1743.750	1946.155
6	999.248	1184.455	1440.134	1765.415	1957.581
7	1019.052	1184.608	1467.366	1766.361	1987.556
8	1031.837	1225.375	1487.512	1825.602	2021.640
9	1023.229	1224.420	1479.268	1824.121	2013.354
10	1053.974	1253.610	1519.011	1866.665	2031.377
Mean	994.469	1172.521	1433.278	1747.780	1943.543
STD	38.877	53.840	56.771	79.232	72.908

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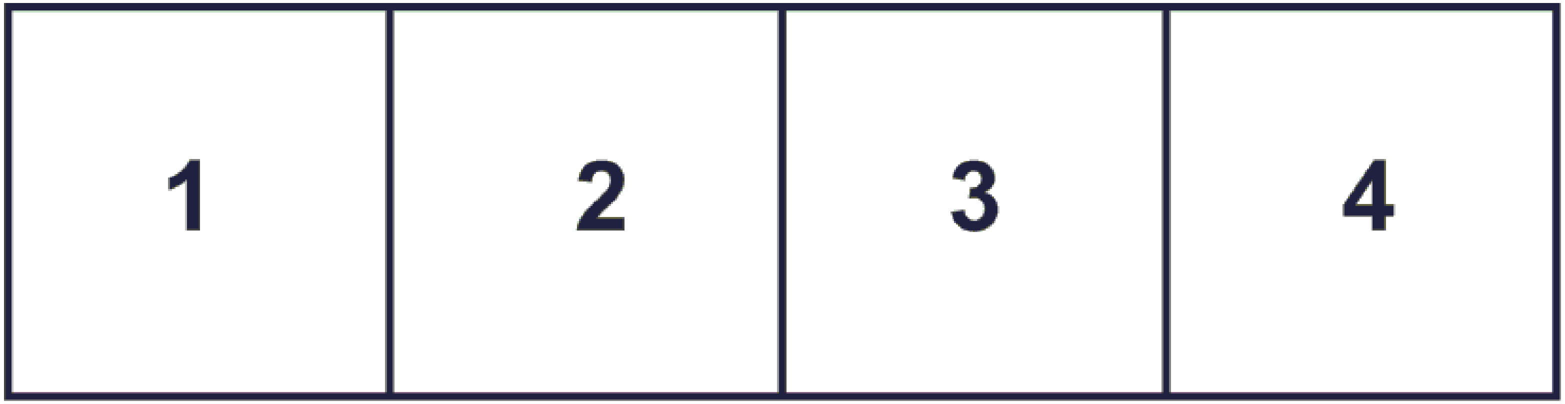
Parameterisation into Four Regions



Initial parameters

$$\bar{t}_i = 4\,\mathrm{mm}, \quad \mathrm{std}(t_i) = 0.8\,\mathrm{mm}, \quad i = 1,\ldots,4.$$

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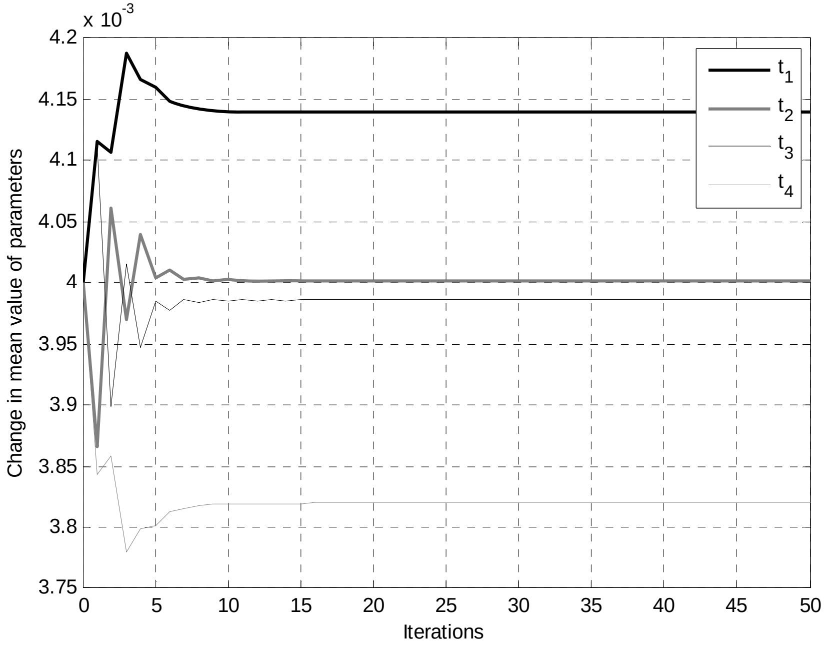
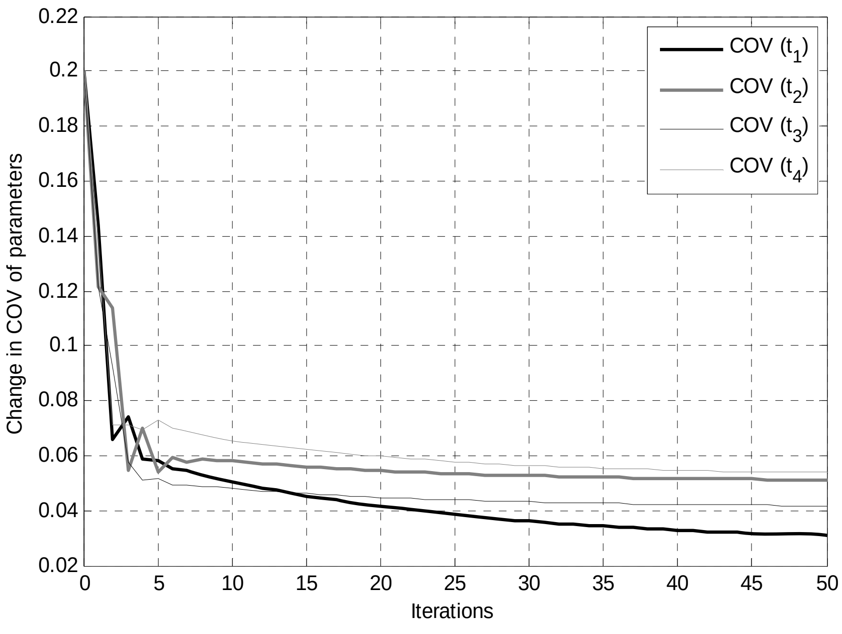
Convergence of Parameter Estimates





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Measured, Initial and Updated Mean and Standard Deviation of Parameters

mm	Measured Parameters	Initial Parameters	Updated Parameters	Initial FE % error	Updated FE % error
$\bar{t}_1$	3.978	4.000	4.140	0.553	4.072
$\mathrm{std}(t_1)$	0.159	0.8	0.129	403.145	-18.868
$\bar{t}_2$	3.969	4.000	4.002	0.781	0.831
$\mathrm{std}(t_2)$	0.161	0.8	0.204	396.894	26.708
$\bar{t}_3$	3.982	4.000	3.986	0.452	0.100
$\mathrm{std}(t_3)$	0.164	0.8	0.166	387.805	1.219
$\bar{t}_4$	3.981	4.000	3.820	0.477	-4.044
$\mathrm{std}(t_4)$	0.167	0.8	0.206	379.042	23.353

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Measured, initial and updated mean natural frequencies

	Measured (Hz)	Initial FE (Hz)	Updated FE (Hz)	Initial FE % error	Updated FE % error
Mode (1)	128.720	128.321	128.111	-0.310	-0.473
Mode (2)	303.177	307.147	306.339	1.310	1.043
Mode (3)	357.597	356.645	355.185	-0.266	-0.675
Mode (4)	630.033	637.433	633.188	1.175	0.501
Mode (5)	705.771	705.467	701.777	-0.043	-0.566
Mode (6)	994.469	1002.229	996.865	0.780	0.241

Measured, initial and updated std of natural frequencies

	Measured (Hz)	Initial FE (Hz)	Updated FE (Hz)	Initial FE % error	Updated FE % error
Mode (1)	6.011	20.943	5.750	248.411	-4.342
Mode (2)	12.032	47.385	13.777	293.825	14.503
Mode (3)	17.048	39.231	15.180	130.121	-10.957
Mode (4)	25.235	65.655	26.797	160.175	6.190
Mode (5)	32.854	71.379	28.644	117.261	-12.814
Mode (6)	38.877	108.445	40.166	178.944	3.316