A Reduced-Order Model for Two-sided Interfaces

D. Dane Quinn[1] Jason Miller[1]

[1]The University of Akron
Akron, OH 44325–3903

April 27–29 2009

special thanks to Dan Segalman and Sandia National Labs

Abstract

Modeling mechanical joints in an accurate and computationally efficient manner is of great importance in the analysis of structural systems, which can be composed of a large number of connected components. This work presents an interface model that can be decomposed into a series-series Iwan model together with an elastic chain, subject to interfacial shear loads. The model is developed and two formulations of the model are considered. Results are then presented as the interface is subject to harmonic loading of varying amplitude. The models presented are able to qualitatively reproduce experimentally observed dissipation scalings. Finally, the interface models are embedded within a larger structural system to illustrate there effectiveness in capturing the structural damping induced by mechanical joints.

- Friction damping in mechanical joints and interfaces contribute a significant fraction of the dissipation in complex engineering structures;

What do we need? **Better Physics, Better Computations**

- Predictive structural models require an accurate representation of the behavior at and near the interface;
- **Small length scales of microslip lead to prohibitivly large computational times**

Existing approach:

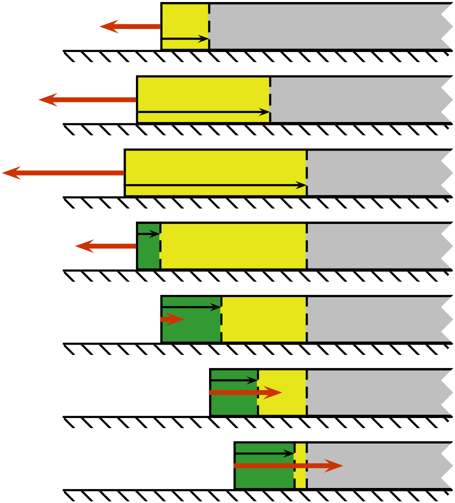
- incorporate the observed dissipation into a linear joint model with effective mass, damping and stiffness parameters

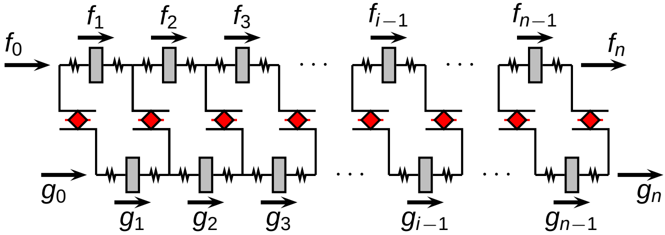
But the tuning is tied to the response of a particular test—the joint model is no longer predictive.

- Menq et al. (1986a,b) develop a continuum model representing the microslip that arises in frictional dampers;
- Segalman (2002) has developed a four parameter Iwan model that is capable of reproducing the qualitative properties of the joint dynamics;
- Song et al. (2004, 2002) have developed an adjusted Iwan beam element (AIBE) based on a parallel-series Iwan model that can be naturally incorporated into an existing finite element framework. With the proper identification of the model parameters, the AIBE can be used to capture experimentally observed profiles for the response of jointed structures.

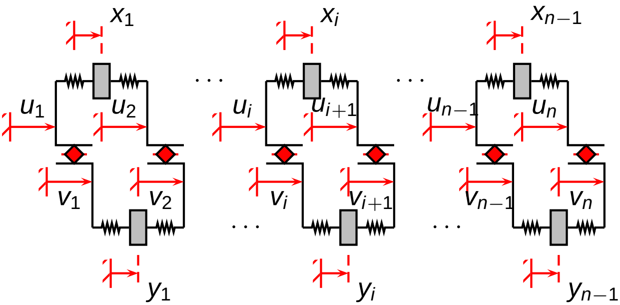
As the loading evolves, multiple slip intervals are developed. . .

slip interfaces initiate at force reversals and move into the material from the free end





- each element is assumed to be identical, with a mass m, and a stiffness k respectively;
- f_i and g_i, $i = 1, \ldots, n-1$ represent the shear loading applied to the masses;
- f_0 and g_0 (f_n and g_n) describe the forces acting on the left (right) edge of the interface
- each interface is described through the frictional force σ_i.



From the the symmetry of the system, the following coordinates can be identified

$$w_i = \frac{x_i + y_i}{2}, \qquad z_i = \frac{x_i - y_i}{2}, \qquad p_i = \frac{u_i + v_i}{2}, \qquad q_i = \frac{u_i - v_i}{2},$$

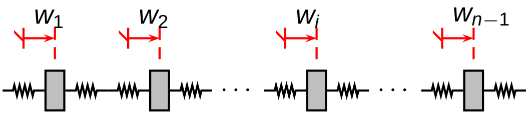
The equations on w_i can be reduced to

$$m\,\ddot{w}_1 + \frac{k}{2}\left(w_1 - w_2\right) = \left(\frac{f_0 + g_0}{2}\right) + \left(\frac{f_1 + g_1}{2}\right),$$

$$m\,\ddot{w}_i + \frac{k}{2}\left(-w_{i-1} + 2\,w_i - w_{i+1}\right) = \left(\frac{f_i + g_1}{2}\right) \qquad i = 2, \ldots, n-2$$

$$m\,\ddot{w}_{n-1} + \frac{k}{2}\left(-w_{n-2} + w_{n-1}\right) = \left(\frac{f_{n-1} + g_{n-1}}{2}\right) + \left(\frac{f_n + g_n}{2}\right).$$

Equivalent to the response of an elastic chain



The dissipation in the system arises solely from the equations on z_i

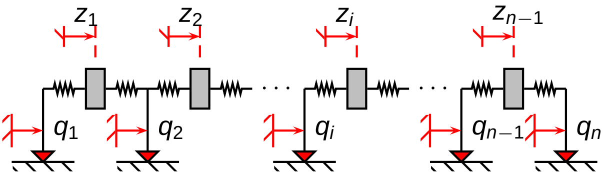
$$m\,\ddot{z}_i + k\left((z_i - q_i) + (z_i - q_{i+1})\right) = \frac{f_i - g_i}{2},$$

with

$$\sigma_1 = -\left(\frac{f_0 - g_0}{2}\right) - k\,(z_1 - q_1),$$

$$\sigma_i = -k\,\left((z_{i-1} - q_i) + (z_i - q_i)\right) \qquad i = 2, \dots, n-1,$$

$$\sigma_n = -\left(\frac{f_n - g_n}{2}\right) - k\,(z_{n-1} - q_n).$$



If the overall static stiffness and total mass of the chain are held fixed as K_{eq} and M_{eq} respectively, then the inter-element stiffness and mass can be represented as

$$k = (n-1)\,K_{\text{eq}}, \qquad\qquad m = \frac{M_{\text{eq}}}{2\,(n-1)}.$$

If the lowest characteristic frequency of the interface scales as $\omega_{\text{c}} = \sqrt{K_{\text{eq}}/M_{\text{eq}}}$, then the largest characteristic frequency scales as

$$\omega_{\text{max}} = \sqrt{\frac{2\,k}{m}} = 2\,(n-1)\sqrt{\frac{K_{\text{eq}}}{M_{\text{eq}}}} = 2\,(n-1)\,\omega_{\text{c}}.$$

Approximate the response of the elastic chain with a Galerkin method

- keep only the linear vibrational modes whose characteristic times are comparable to the timescales of the surrounding structure;
- depends on the structure as well as the forcing—longer timescales require, in general, fewer modes.

Retaining only the lowest s linear modes for the elastic chain, denoted as ϕ_i, $i = 1, \ldots, s$, the response of this component is then given as

$$w_i(t) = \sum_{j=1}^{s} W_j(t) \, [\phi_j]_i,$$

where

$$\hat{\boldsymbol{M}} \, \ddot{\boldsymbol{W}} + \hat{\boldsymbol{K}} \, \boldsymbol{W} = \hat{\boldsymbol{f}}(t),$$

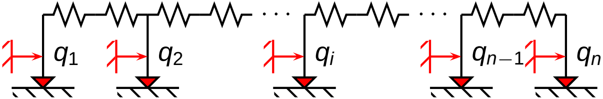
$$[\hat{\boldsymbol{M}}]_{jk} = \phi_j^T \boldsymbol{M} \phi_k, \quad [\hat{\boldsymbol{K}}]_{jk} = \phi_j^T \boldsymbol{K} \phi_k, \quad [\hat{\boldsymbol{f}}(t)]_j = \phi_j^T \boldsymbol{f}(t).$$

In the series-series Iwan chain, the dissipation can be accurately captured by neglecting the mass in each Iwan element—effectively solving for the quasistatic response (Kim and Kwak, 1996; Berger et al., 2000; Cocu et al., 1996)

$$k\ (q_1 - q_2) = (f_0 - g_0) + \left(\frac{f_1 - g_1}{2}\right) + 2\,\sigma_1,$$

$$k\ (-q_{i-1} + 2\,q_i - q_{i+1}) = \left(\frac{f_{i-1} - g_{i-1}}{2}\right) + \left(\frac{f_i - g_i}{2}\right) + 2\,\sigma_i \qquad i = 2, \ldots$$

$$k\ (-q_{n-1} + q_n) = (f_n - g_n) + \left(\frac{f_{n-1} - g_{n-1}}{2}\right) + 2\,\sigma_n,$$



Direct time-dependent loading

$$f_0(t) = 0, \qquad f_n(t) = F_0 \sin(\omega t),$$
$$g_0(t) = -F_0 \sin(\omega t), \qquad g_n(t) = 0.$$

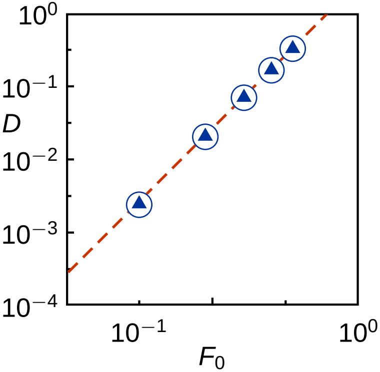
The relative displacement across the interface

$$\Delta_1 = u_n - v_1 = \left(w_{n-1} - w_1\right) + \left(q_n + q_1\right) + \left[\frac{(f_n + g_n) - (f_0 + g_0)}{2\,k}\right],$$

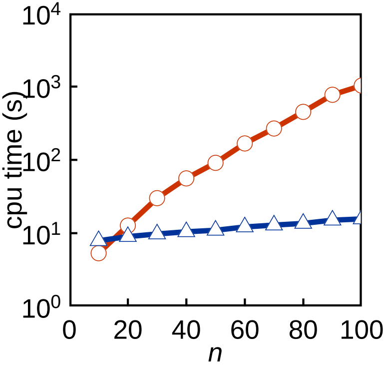
The mass and stiffness are chosen as

$$k = (n-1), \qquad m = \frac{1}{2\,(n-1)},$$

so that the total mass and overall equivalent stiffness for the interface model are unity, that $K_{\text{eq}} = 1$ and $M_{\text{eq}} = 1$.

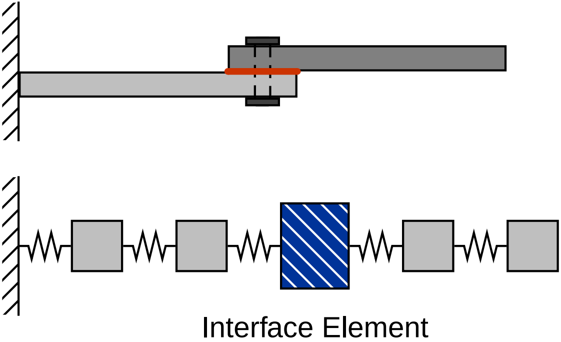


$n = 40$, $\omega = \pi/15$
circles: original
triangles: reduced
dashed: continuum
$D = 8/3\,(F_0)^3$



$F_0 = 0.30$, $\omega = \pi/15$
circles: original
triangles: reduced

Incorporate the interface model into an elastic beam



The ℓ^{th} mass is replaced by an Iwan interface element (Song et al., 2004).

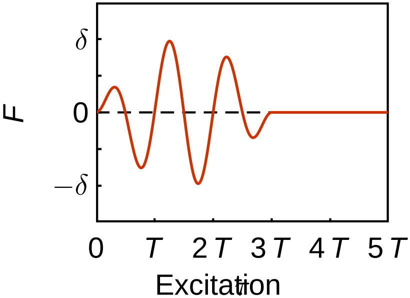
Near the interface the equations governing $a_{\ell-1}$ and $a_{\ell+1}$ become

$$\begin{aligned}\frac{M}{r}\,\ddot{a}_{\ell-1} + K\,r\,\left(a_{\ell-1} - a_{\ell-2}\right) &= -g_0(t),\\ \frac{M}{r}\,\ddot{a}_{\ell+1} + K\,r\,\left(-a_{\ell+2} + a_{\ell+1}\right) &= -f_n(t),\end{aligned}$$

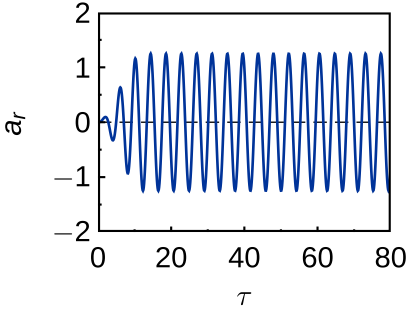
where $g_0(t)$ and $f_n(t)$ represent the coupling between the Iwan interface element and the surrounding chain. These forces are described as

$$\begin{aligned}g_0(t) &= 2\,K\,r\,(a_{\ell-1} - v_1) = \frac{2\,K\,r}{\left(1 + \frac{1}{n-1}\right)}\,\left(a_{\ell-1} - (w_1 - q_1)\right),\\ f_n(t) &= 2\,K\,r\,(a_{\ell+1} - u_n) = \frac{2\,K\,r}{\left(1 + \frac{1}{n-1}\right)}\,\left(a_{\ell-1} - (w_1 + q_n)\right),\end{aligned}$$

and $f_0(t) = g_n(t) \equiv 0$. The quantities w_1, q_1, w_{n-1}, and q_n represent the elastic and dissipative coordinates used within the interface model described above, and are valid when considering either the conventional, or the massless Iwan interface.

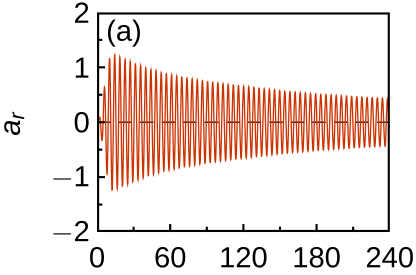


Excitation

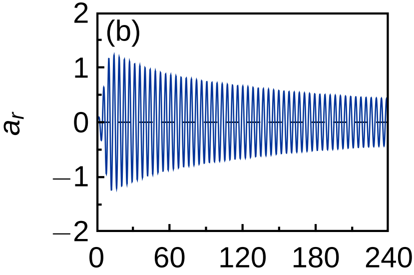


Monolithic structure

$\delta = 0.25$, $T = 4$ ($r = 10$, $j = 6$, $n = 20$)



Conventional Iwan interface



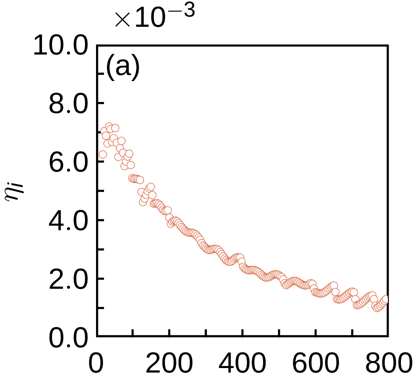
Reduced-order Iwan interface

$s = 4$

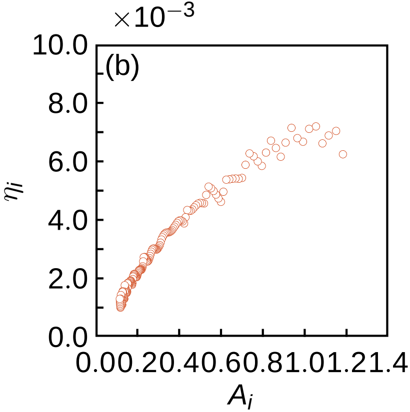
$\delta = 0.25$, $\mu = 1$, $T = 4$ ($r = 10$, $j = 6$, $n = 20$)

Exponential Decay Rate

$$\sigma(t_i) = \frac{\ln(A_{i+1}) - \ln(A_{i-1})}{t_{i+1} - t_{i-1}}.$$

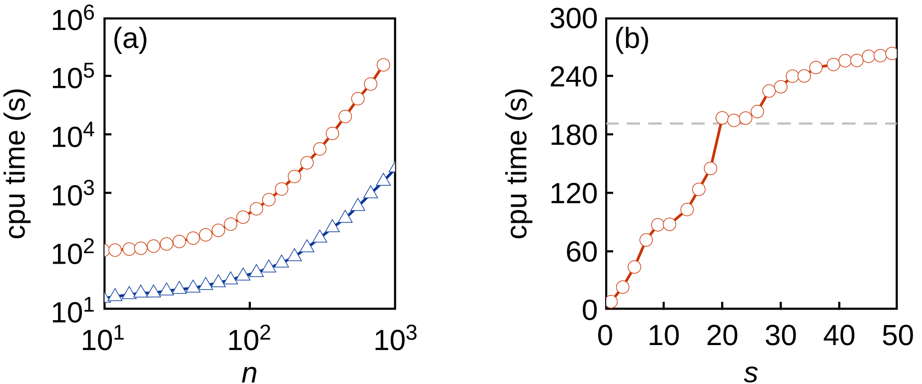


σ vs. τ t



σ vs. A

Exponential decay rate σ as identified from simulation ($r = 10$, $j = 6$;



$s = 4$ $n = 50$

Computational time required to simulate 400 time units ($r = 10$, $j = 6$, $\delta = 0.25$, $\mu = 1.00$)

- Incorporate this model into a finite element formulation;
- Determine the appropriate loadings on this interface model.

Berger, E. J., Begley, M. R., Mahajani, M., 2000. Structural dynamic effects on interface response: Formulation and simulation under partial slipping conditions. J. Appl. Mech.—T. ASME 67, 785–792.

Cocu, M., Pratt, E., Raous, M., 1996. Formulation and approximation of quasistatic frictional contact. Intl. J. Engr. Sci. 34 (7), 783–798.

Kim, J., Kwak, B., 1996. Dynamic analysis of two-dimensional frictional contact by linear complementarity problem formulation. Intl. J. Solids Struct. 33, 4605–4624.

Menq, C.-H., Bielak, J., Griffin, J. H., 1986a. The influence of microslip on vibratory response, part I: A new microslip model. J. Sound Vib. 107 (2), 279–293.

Menq, C.-H., Griffin, J. H., Bielak, J., 1986b. The influence of microslip on vibratory response, part II: A comparison with experimental results. J. Sound Vib. 107 (2), 295–307.

Segalman, D. J., 2002. A four-parameter Iwan model for lap-type joints. Tech. Rep. SAND2002–3828, Sandia National Laboratories.

Song, Y., Hartwigsen, C. J., Bergman, L. A., Vakakis, A. F., 2002. A phenomenological model of a bolted joint. Presented at the 15th