Nonlinear System Identification: Challenges and Open Questions

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Nonlinear System Identification: Overview

- The difficulty in developing NSI methodologies that are valid for broad classes of dynamical systems is due to the well-recognized highly individualistic nature of nonlinear systems which restricts the unifying dynamical features that are amenable to system identification
- Some limitations of current methods:
  - Weak nonlinearities
  - Relatively simple systems (i.e., one or two-DOF)
  - Computationally intensive
  - No general methods for systems with non-smooth characteristics (e.g., clearances, friction), strong nonlinearities (e.g., ultra-flexible wings)

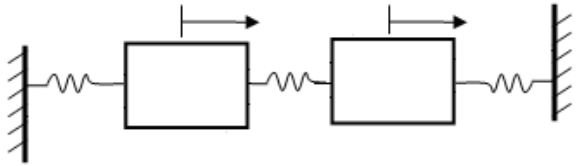
Nonlinear System Identification: Challenges

- One of the challenges of NSI is the well known sensitivity of nonlinear systems to initial and forcing conditions
- So even the simple task of identifying a set of (linearized) modal matrices modified ('perturbed') by nonlinear corrections might be an oversimplification of the problem.
- Effects of non-smooth effects on the dynamics?
- How to address multi-physics and multi-scale effects in NSI?
- How to tie uncertainty modeling with NSI?
- How to identify strongly nonlinear complex nonlinear modal (resonance) interactions?
- Effects of middle- or high-frequency dynamics?

Nonlinear System Identification: Preliminaries

- Simple two-DOF system:

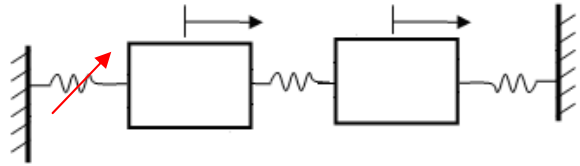
$$\ddot{x}_1 + (2x_1 - x_2) = 0$$
$$\ddot{x}_2 + (2x_2 - x_1) = 0$$



Nonlinear System Identification: Preliminaries

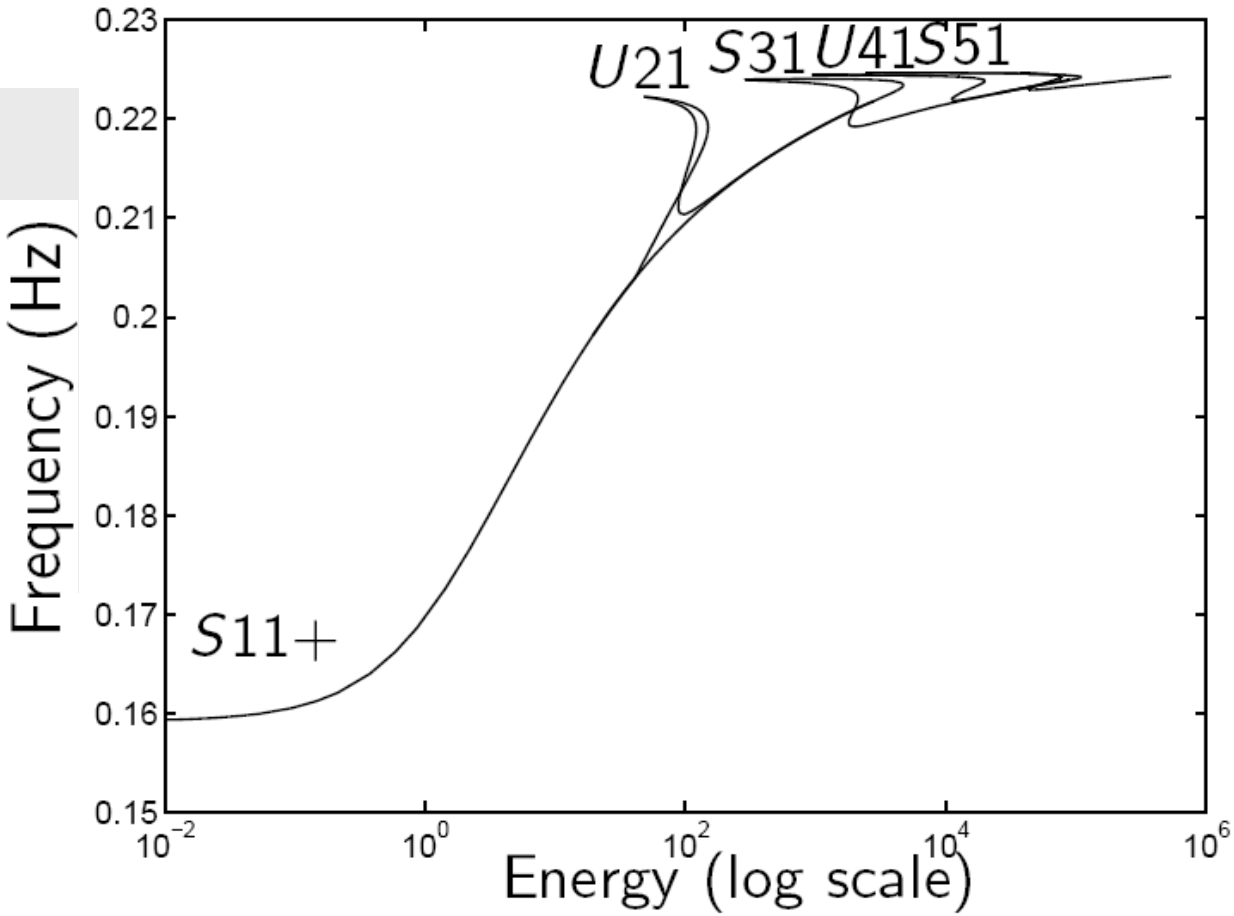
- Now we add a simple (and common) cubic nonlinearity:

$$\ddot{x}_1 + (2x_1 - x_2) + 0.5x_1^3 = 0$$
$$\ddot{x}_2 + (2x_2 - x_1) = 0$$



Nonlinear System Identification: Preliminaries

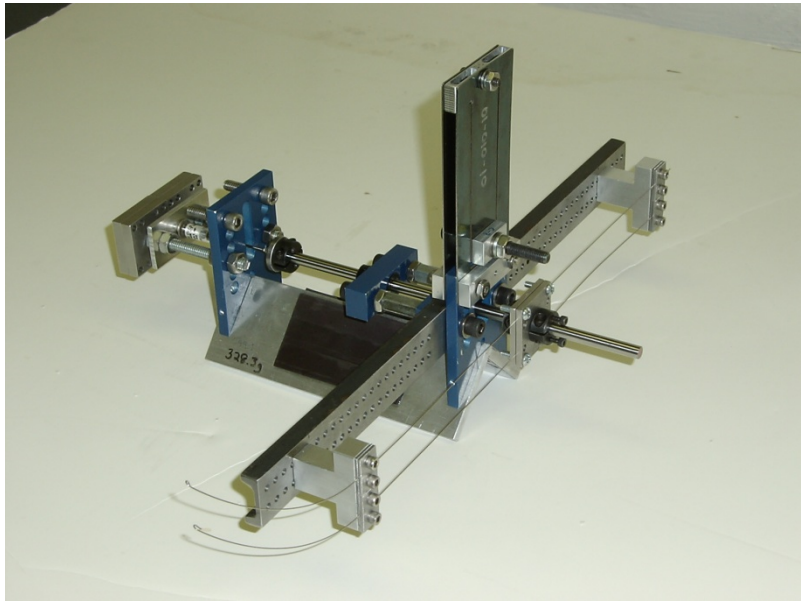
High-frequency strongly nonlinear modal interactions are unavoidable!

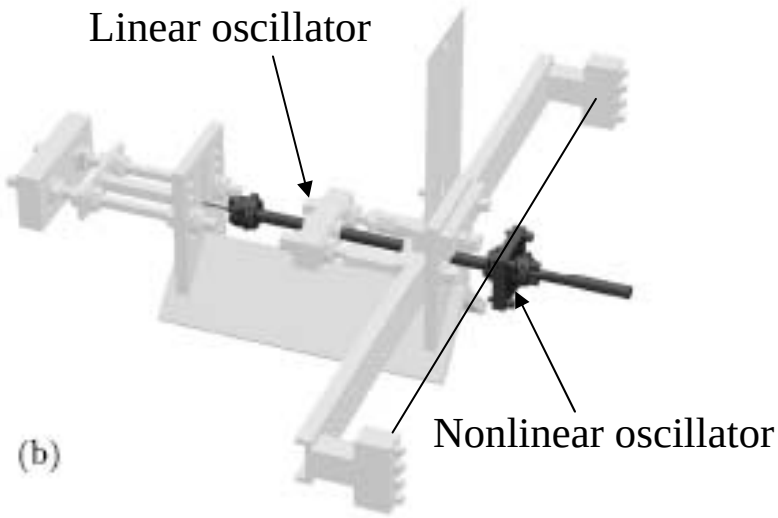


Nonlinear System Identification: Preliminaries

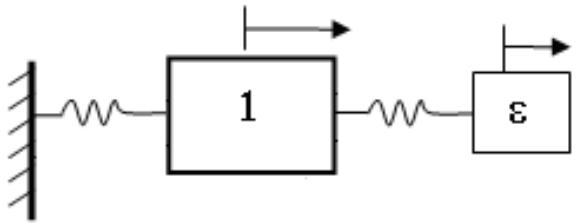
- A very useful feature of the FEP is its relation to the transient dynamics of the corresponding weakly damped system.
- This is due to the fact that the effect on the dynamics of weak damping is parasitic: Instead of introducing 'new' dynamics, it just causes transitions of the dynamics between branches of normal modes leading to multi-frequency nonlinear dynamical transitions.
- Different initial or forcing conditions may lead to drastically different transitions in the FEP. We demonstrate this by a simple two-DOF nonlinear system.

Nonlinear System Identification: Example 1



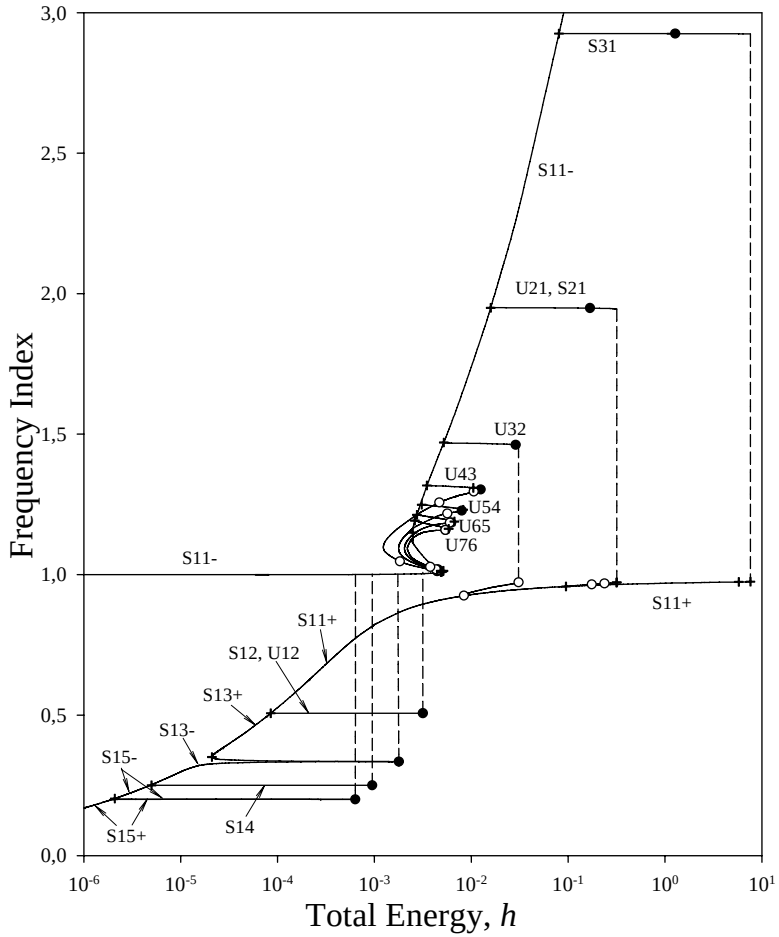


$$\ddot{x}_1 + \omega_n x_1 + C(x_1 - x_2)^3 = 0$$
$$\varepsilon \ddot{x}_2 + C(x_2 - x_1)^3 = 0$$

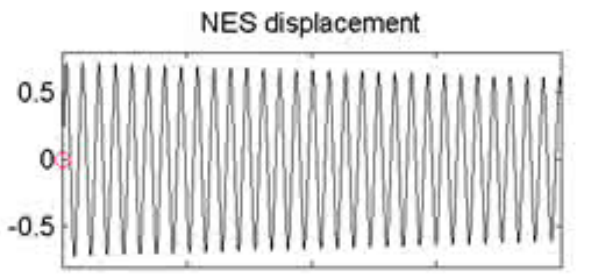


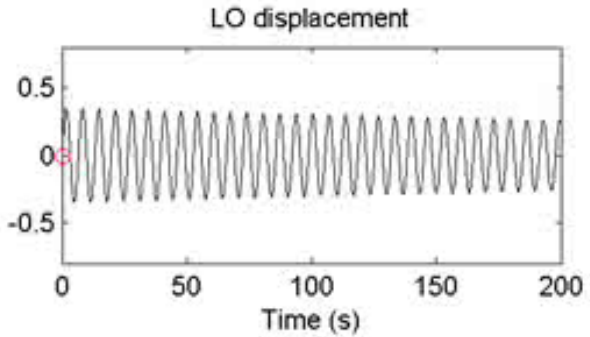
Nonlinear System Identification: Example 1

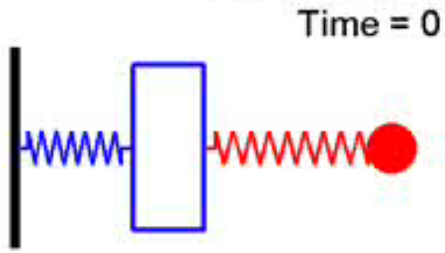
- Very complicated structure of nonlinear modes in the FEP for this two-DOF system!

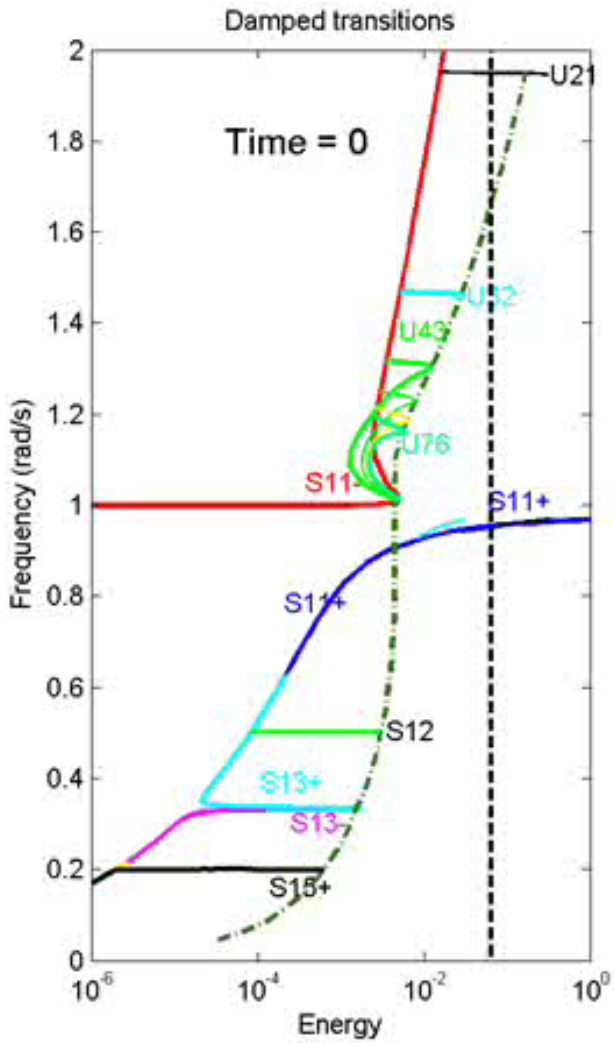


Nonlinear System Identification: Sensitivity on ICs

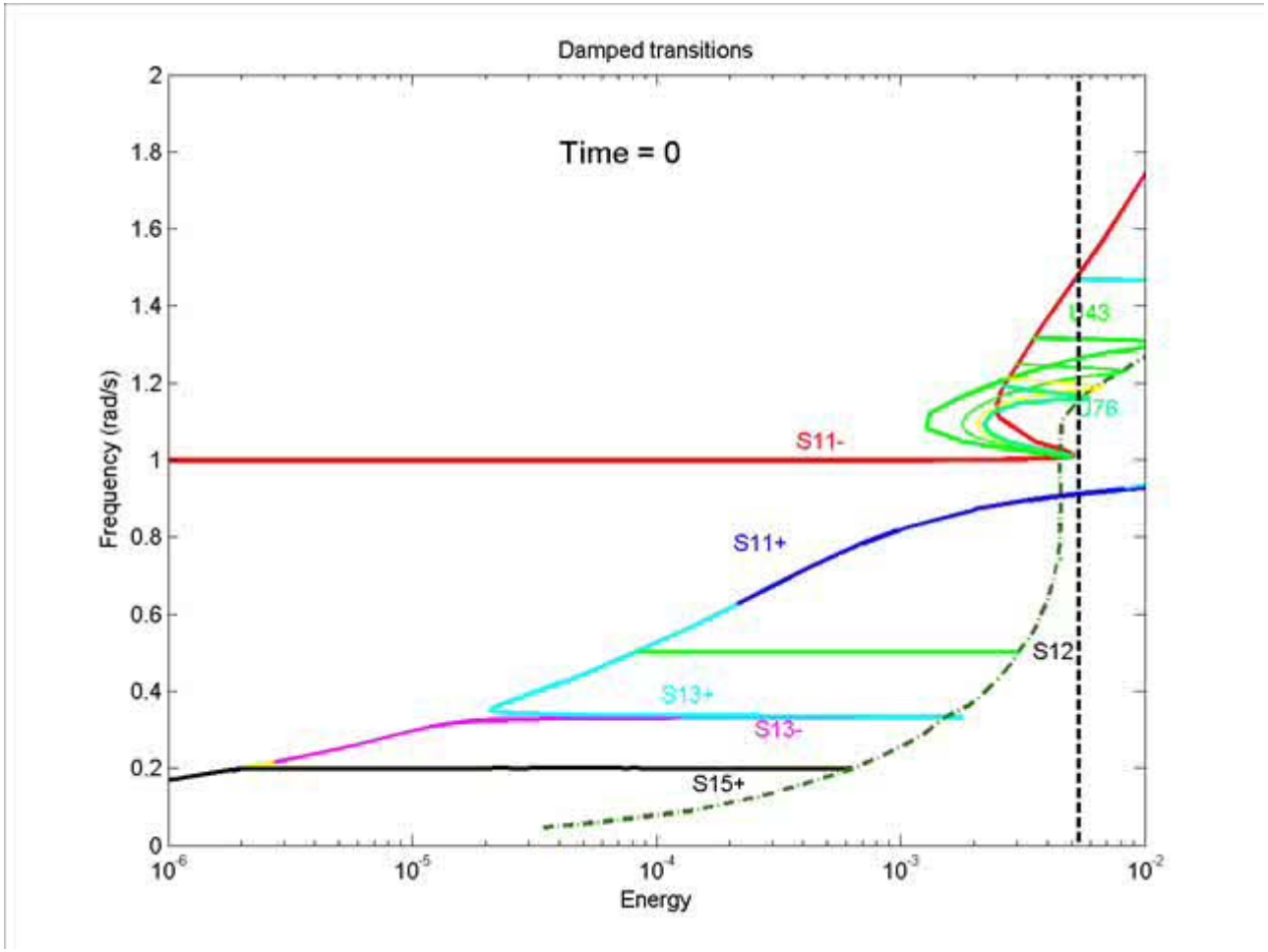








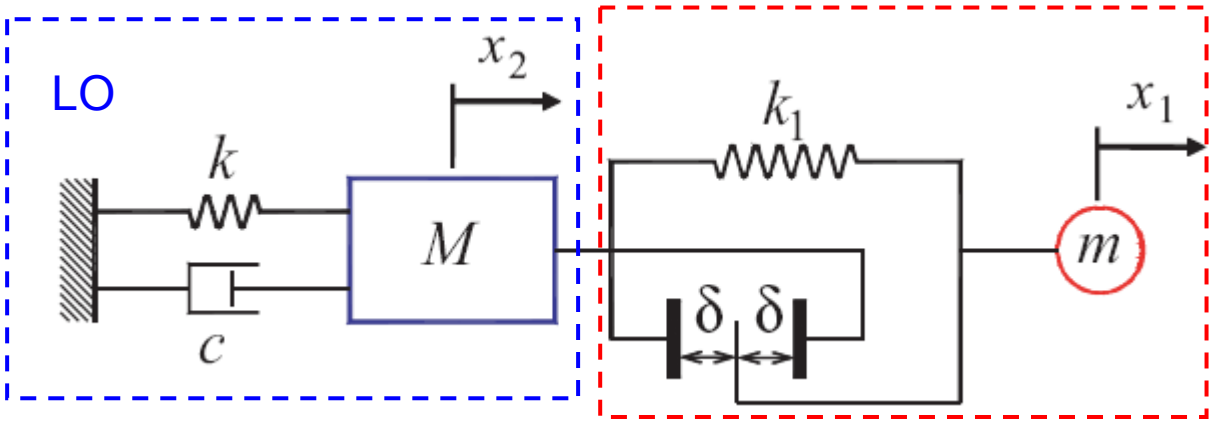
Nonlinear System Identification: Sensitivity on ICs



Nonlinear System Identification: Challenges

- The previous highlight the important challenges that the analyst is faced with when performing NSI:
  - First, to address the (generic) feature of nonlinear systems to exhibit qualitatively different responses with varying energy and/or initial conditions → need to adopt a global approach for identifying the dynamics over broad frequency and energy ranges.
  - Second, identify complex multi-frequency transitions for particular initial conditions → dictates a local approach, whereby a specific nonlinear transition is considered and the task is to identify the nonlinear modal interactions that govern this transition.

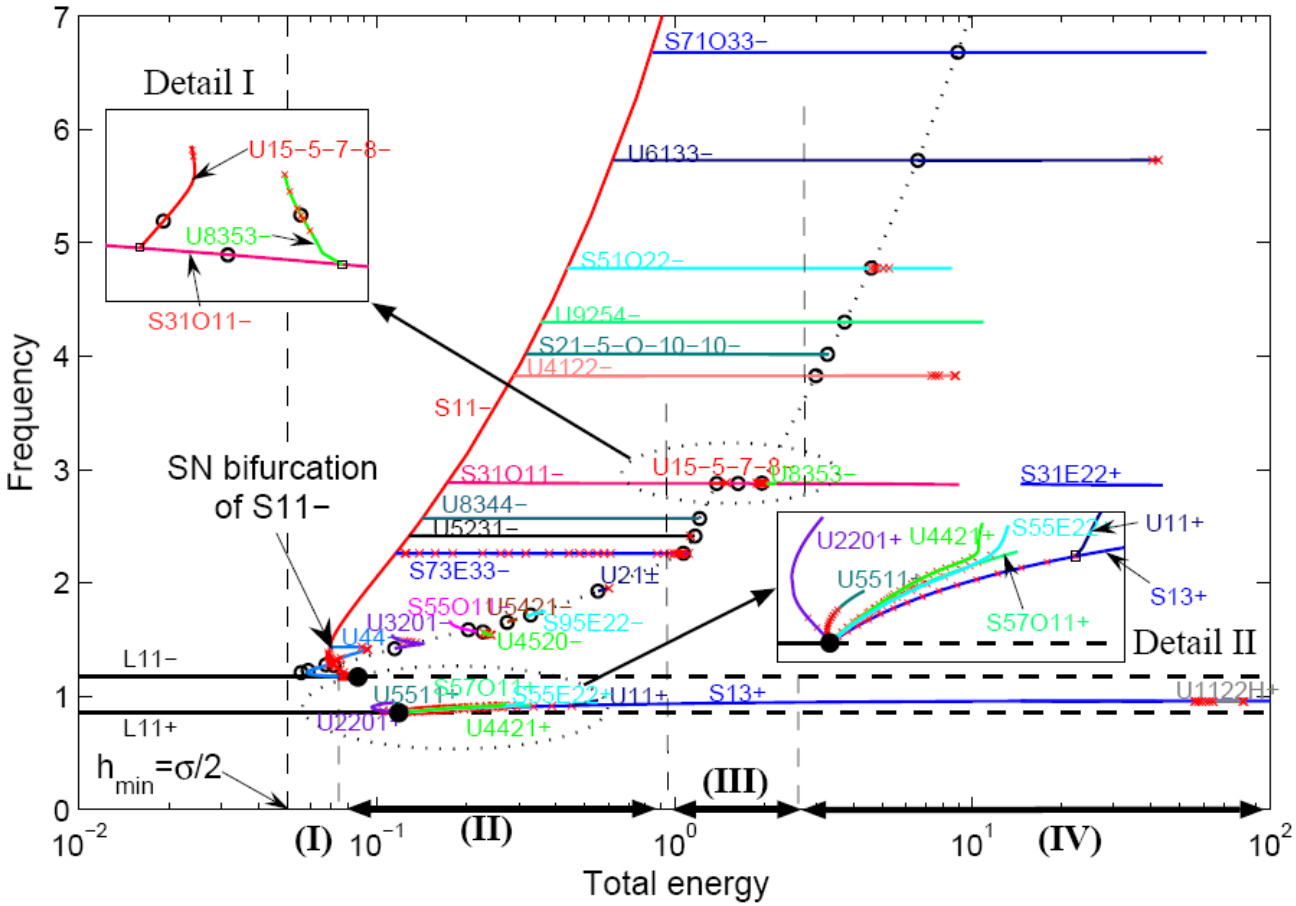
Example 2: Joint with clearance



$| x_1 - x_2 | < \delta \Rightarrow$ Vibrations of 2-DOF linear oscillator

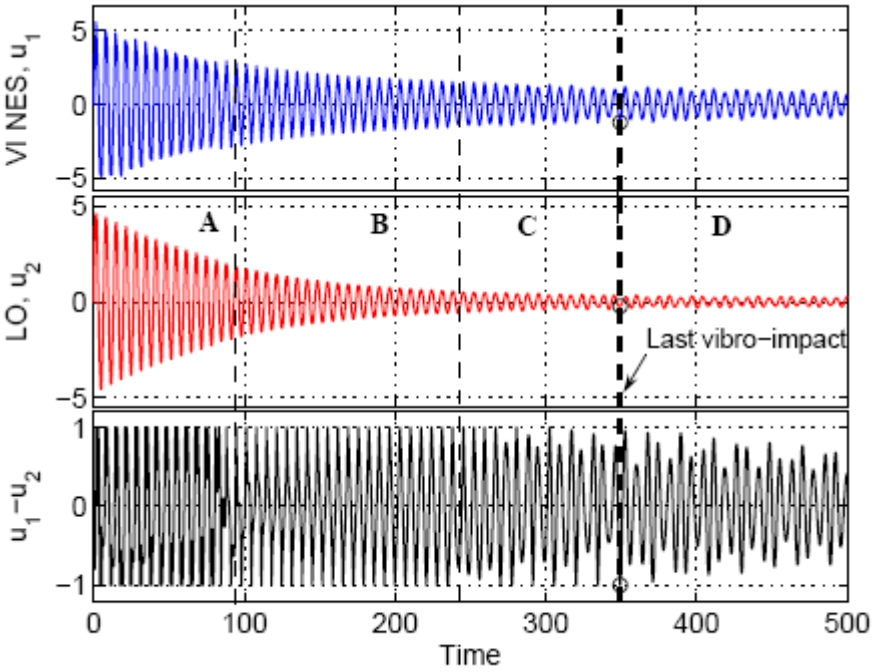
$| x_1 - x_2 | = \delta \Rightarrow$ Vibro-impacts occur

Nonlinear System Identification: Example 2



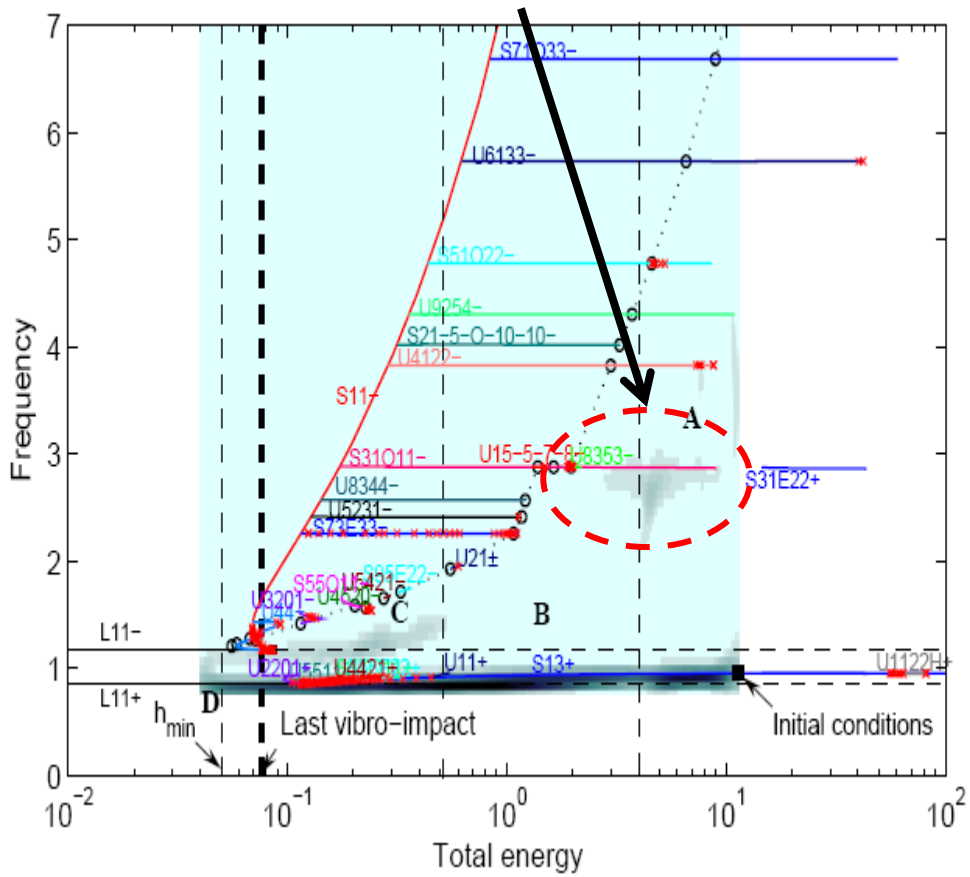
The FEP shows that the system possesses a very complicated topological structure of dynamics due to the strong vibro-impact nonlinearity.

Nonlinear System Identification: Example 2



The initial conditions at 1:1 in-phase NNM are imposed on the system; that is, the resulting VI responses at the initial transient should exhibit 1:1 transient resonance capture.

Does this portion of VI damped transitions on FEP represent superharmonic resonant interactions, instead of 1:1 resonance?



NSI Methodology: Major Challenges

- Global / local aspects of dynamics need to be addressed
- Need for ROMs that take into account the generic sensitivity of nonlinear systems to changes in initial and forcing conditions.
- Capacity for separating smooth from non-smooth effects, identifying both (especially friction and clearance effects)
- Based on direct analysis of response time series?
- Natural nonlinear extension of linear modal analysis?
- Data driven ROMs in multi-physics problems?
- Probe middle / high frequency dynamics coupling vibrations- and wave-based approaches?
- …